Development of Transition Edge Sensor Distributed Read-Out Imaging Devices for Applications in X-ray Astronomy

Thesis submitted for the degree of Doctor of Philosophy at the University of Leicester

By

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Abstract

This thesis is concerned with the development of, position-sensitive, Transition Edge Sensors (TESs) operating at cryogenic temperatures (~ 0.1 K). The Distributed Read-Out Imaging Device (DROID) uses TES read-out at both ends of a linear X-ray absorber, to derive, through heat diffusion, both spectral and spatial information. Potentially, DROIDs offer a simpler technological alternative to the development of large area pixel arrays for future Xray space observatories. We have established a finite-element model to numerically calculate the response of the DROID to an X-ray photon. The model estimates the noise spectral density at the detector outputs, including the frequency dependent correlations between the two TESs. This model is used to implement pre-existing signal processing algorithms, based on the digital optimal filter, to calculate the position and energy resolution along the length of experimental DROID designs. We show that these algorithms do not lead to optimum performance under all conditions and derive the true optimal filters, based on least-squares minimisation of the total noise power in the DROID. By numerical simulation, we show that improvements in the energy and in particular, in the position resolution, are theoretically We investigate the trade-offs resulting from changing key detector design possible. parameters, such as the thermal conductances of the different detector elements. These simulations enable the DROID design to be optimised for specific detector applications. The design and experimental characterisation of prototype DROIDs are described. The first X-ray results from a prototype DROID, using single TES read-out, are reported. The data shows different populations of signal corresponding to X-ray absorption in different parts of the DROID. These results demonstrate proof of concept, confirming spatial sensitivity along the length of the DROID absorber, though the actual spectral and spatial resolutions are limited by the availability of only a single read-out channel.

Declaration

I hereby declare that no part of this thesis has been previously submitted to this or any other University as part of the requirement for a higher degree. The work described herein was conducted solely by the undersigned except for those colleagues and other workers acknowledged in the text.

Stephen James Smith

24th April 2006

Publications

Some of the results in this thesis are reported in the following publications:

Smith S. J., Whitford, C., Fraser G. W., Holland A. D., Goldie D., Ashton T. J. R., Limpenny R. J., Stevenson T., First results from a 1-D imaging spectrometer using Ir TESs, Nucl. Instr. and Meth. A 520 (2004) 449.

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Chapter 1: Introduction and Background

This Chapter gives an introduction to the field of X-ray astronomy, and describes the motivation behind the research carried out in this Ph.D. thesis. The next generation of space based X-ray telescope is discussed as well as the need for using cryogenically-cooled focal plane detectors. Finally, an outline of the thesis structure is presented.

1.1. The X-ray Universe

Over the past 40 years, since the discovery of the first cosmic X-ray sources [Giacconi et al., 1962], Xray astronomy has developed into one of the most important fields in astrophysics. Hot plasmas with temperatures of 10⁶-10⁸ K are found in many objects in the Universe such as accreting binaries, stellar coronae, black holes, clusters of galaxies, and active galactic nuclei (AGN) [Charles and Seward, 1995]. X-ray spectroscopy provides a means of studying both the physical and dynamical properties of matter under extreme conditions of gravity, temperature and density, giving insights into the structure and evolution of the Universe and matter within it.

The water content in the Earth's atmosphere absorbs electromagnetic radiation in the X-ray band; hence the detection of X-rays can only be made from high altitude. More penetrating hard X-rays (> 20 keV) are detectable by balloon-borne experiments at altitudes of 50-100 km, whereas less penetrating soft X-rays (0.1-20 keV) require sounding rockets or satellite-based instruments reaching altitudes of greater than 200 km. Consequently, it was not until the development of sounding rockets in the late 1940s that high altitude space based measurements could become a reality. The first solar X-rays from space were observed using a captured German V2 rocket [Friedman et al., 1951]. Until the discovery by Giacconi et al. (1962) of the first cosmic source, Sco X-1, it was generally thought that there would be no detectable emission from any normal star, thus there was no interest in looking for cosmic sources. The sounding rocket experiment of 1962 was intended to measure direct solar X-ray emission and the fluorescent emission from the lunar surface. In addition to these measurements, a diffuse X-ray background was discovered as well as the brightest X-ray source in the sky, Sco-X1. The first dedicated X-ray satellite, Uhuru [Giacconi, 1971], was lunched in 1970 and identified over

300 individual X-ray sources, including; X-ray binaries, supernovas, active galaxies, quasars, and clusters of galaxies. This success was followed up by further dedicated satellites such as Ariel 5 (1974) and SAS-3 (1975). More recently, satellites such as EXOSAT (1983), Ginga (1987), ROSAT (1990) and ASCA (1993), and the current state of the art X-ray observatories Chandra (1999) [Weisskopf et al., 2000], XMM-Newton (1999) [Jansen et al., 2001] and Astro-E2 (2005) [Inoue, 2003], have offered significant improvements in collecting power, spectral and angular resolution, resulting in continued advances in the understanding of the nature of the X-ray sources and the mechanism by which their fluxes are generated.

1.2. The XEUS Mission and Instrument Requirements

The X-ray Evolving Universe Spectrometer or XEUS [Parmar et al., 2004] is an ambitious next generation space based X-ray observatory, currently under study by the European Space Agency (ESA) and the Japanese Aerospace Exploration Agency (JAXA), and is a follow on to the XMM-Newton cornerstone. A similar mission, Constellation-X [White et al., 1999], is under study by National Aeronautic and Space Administration (NASA), though it is possible that the XEUS and Constellation-X concepts will be combined into a single joint ESA/NASA/JAXA mission.

The case for the XEUS mission is driven by the astrophysics science requirements arising from the successful XMM-Newton/Chandra era. The main focus is on the study of hot matter in the early Universe, particularly the formation and evolution of objects such as black holes and galaxy clusters [Parmar et al., 2004]. The measurement of these early objects in the Universe requires a minimum detectable flux of $< 10^{-17}$ ergs cm⁻² s⁻¹, which is ~ 200 times more sensitive than XMM-Newton and a factor of 10 better than the limiting sensitivity on Chandra. To achieve such sensitivities, the XEUS telescope requires a large effective area and a long focal length. The XEUS mirrors [Bavdaz et al., 2004a] are of a grazing incidence Wolter I design and will have a diameter of 10 m compared to 0.7 m for the outer shells of the XMM-Newton mirror modules, making an effective area of $> 20 \text{ m}^2$ at 1 keV (~ 20 times greater than XMM-Newton), with an angular resolution of 2-5 arc seconds (~ 3 times better than XMM-Newton) [Beijersbergen et al., 2004]. The efficiency of Wolter I optics increases with decreasing grazing angle (between the X-rays and the mirrors). Consequently, to achieve the necessary minimum detectable flux level, a long focal length is required. To meet this requirement the XEUS concept incorporates the novel design of placing the detectors and optics on separate spacecraft (detector spacecraft (DSC) and mirror spacecraft (MSC)), making it possible to achieve a 50 m focal length, which would otherwise be impractical to construct as one solid or deployable structure. The two spacecraft then fly in formation to an accuracy of $< 1 \text{ mm}^3$ [Bavdaz et al., 2004b]. Both the DSC and MSC will be launched separately and rendezvous for autonomous deployment at the second Earth-Sun Lagrangian point (L2), which is situated 1,500,000 km from the Earth (on the night side) with the

same solar orbital period as the Earth. The region about L2 is a gravitational saddle point where spacecraft can remain at an approximately constant distance from the Earth throughout the year using small station-keeping manoeuvres. Thus, the deployment at the L2 point reduces fuel usage, which would otherwise be required by the DSC to track the MSC in Earth orbit and reduce the mission lifetime. Further, a spacecraft orbiting L2 can be positioned to permanently point away from the Sun and Earth, enabling continuous observation, which would not be possible in an Earth orbit.

Parameter	WFI	NFI 1	NFI 2
Detector Type	DEPFET	STJ	TES
Field of View	5×5 arcmin	0.5×0.5 arcmin	0.5×0.5 arcmin
Pixel Size	1×1 arcsec	1×1 arcsec	1×1 arcsec
Energy Range	0.1-15 keV	0.05-2 keV	1-12 keV
Energy Resolution	125 eV at 6 keV	1 eV at 0.5 keV	4 eV at 6 keV
Detection Efficiency	100 % at 1-6 keV	90 % at 1 keV	90 % at 6 keV
Count rate / point spread function	0.2-1 kHz	25 kHz	0.25 kHz

Table 1.1. Expected characteristics of the baseline instruments for XEUS [Lumb D., 2004a].

Three instruments for the XEUS focal plane were originally envisaged to cover the complete set of science requirements [Lumb, 2004b]. These instruments consist of a wide-field imager (WFI) and two narrow field imagers (NFI 1 and NFI 2). The required characteristics of these instruments are summarised in Table 1.1. The WFI is required to cover a large field of view (5×5 arcmin) over a broad energy range (0.1-15 keV) with full width at half maximum (FWHM) energy resolution of 70 eV at 1 keV and 125 eV at 6 keV. Thus the WFI will be used to acquire deep field images containing numerous sources at different flux levels. A DEPFET (DEPleted Field Effect Transistor) pixel array detector has the ability to use different read-out frequencies for different areas of the focal plane and hence handle multiple sources in the field of view at different flux levels and is considered the most suitable candidate for the WFI [Struder, 1999]. The detailed study of selected sources in a small field of view (0.5×0.5 arcmin) will then be carried out by the NFI 1 and NFI 2 instruments. The NFI 1 will cover the lower energies (0.05-2 keV) and NFI 2 the medium energies (1-12 keV), with some overlap in between. To carry out high spectral resolution studies FWHM energy resolutions of < 2 eV at 1 keV and ~ 4 eV at 6 keV are required [Lumb, 2004a].

Currently there are two main detector technologies with the potential capability to achieve these NFI detector requirements, both of which operate in the \sim mK temperature régime [de Korte, 2000].

Superconducting Tunnel Junctions (STJs) [Twerenbold, 1988] are a potential candidate for the NFI 1 instrument. In a STJ, a photon is absorbed in a superconducting layer, breaking apart superconducting electron pairs (Cooper pairs) and creating an excited population of charge carriers known as quasiparticles. These quasiparticles quantum mechanically tunnel through an insulating barrier and are read out as a current pulse on the other side. The initial number of quasiparticles created is proportional to the energy of the incident photon. The achievable energy resolution with such a detector is determined by the statistics of the quasi-particle generation as well as the tunnelling statistics and scales as the square root of the photon energy. This, coupled with low detection efficiency, means that STJs are considered better suited to the lower energy range requirements of the NFI 1 instrument.

Microcalorimeter detectors [Moseley et al., 1984] operating at temperatures of the order 100 mK, such as Transition Edge Sensors (TESs) [Irwin, 1995a] or possibly Metallic Magnetic Calorimeters (MMCs) [Fleischmann et al., 2003] are baselined for the NFI 2 instrument on XEUS [Bruijn et al., 2000]. A microcalorimeter uses a thermometer that measures the temperature increase caused by the absorption of energy from an X-ray photon. In a TES, the rapid change in temperature and hence the resistance, of a thin superconducting layer is used as a measure of the absorbed X-ray energy. In a MMC, a magnetic material is used as the thermometer, whose change in magnetisation is a measure of the temperature change. In contrast to a STJ, for a microcalorimeter the energy resolution is independent of the incident photon energy over the operating range of the detector.

At the University of Leicester we have focused on the development of Transition Edge Sensors for future X-ray astronomy missions such as XEUS. In particular, this thesis has concentrated on the development of a one dimensional, position-sensitive TES based device, known as a Distributed Read-Out Imaging Device (DROID).

1.3. Thesis Structure

Chapters 2-4 of this thesis report a theoretical analysis of DROID and TES performance, starting in Chapter 2 with an outline of the basic physics and design of microcalorimeters including semiconductor thermistors, Metallic Magnetic Calorimeters and Transition Edge Sensors, before a detailed description of TES theory is developed. This fundamental theory is essential in understanding and optimising the performance of the TES. In Chapter 3 we examine the development of position-sensitive TES based detectors, such as arrays of single pixel TESs, outlining the advantages and disadvantages of these compared to large distributed absorber devices such as the Leicester DROID concept. To fully understand the theory behind the DROID concept we require a model capable of describing the response of the device to X-ray photons as well as the intrinsic noise characteristics.

Thus in Chapter 3, we develop a finite-element, numerical model, to mathematically represent the DROID. Using this model we simulate the X-ray response of a current experimental design, as well as the noise spectral density on the detector output. In Chapter 4, the numerical analysis is extended to investigate techniques for determining the energy and position of the incident photon. We apply existing techniques used in similar position-sensitive devices. This leads to the development of optimised algorithms, from which, we show by numerical simulation that improved theoretical energy and position resolution across the length of a DROID can be achieved over the pre-existing methods. We use these algorithms to investigate the DROID design for optimum energy and position resolution performance, based around the optimisation of key DROID design parameters. This detailed theoretical study leads on to experimental results, starting in Chapter 5 with a description of the readout and cooling requirements for the operation of TES detectors. In particular, the characterisation of Superconducting QUantum Interference Devices (SQUIDs) and the associated amplifier chain, used in the experimental programme, is discussed. We also discuss Dilution Refrigerators (DRs) required for TES operation in the mK temperature régime, which were used both at the University of Leicester Space Research Centre and the Cavendish Laboratory University of Cambridge. Chapter 6 starts with a discussion of the requirements of TES absorbers, both for DROIDs and single pixel detectors. In particular we study the respective advantages and disadvantages of metal, dielectric and superconducting absorbers, which are all commonly used as X-ray absorbers. We also describe the design of the current prototype experimental DROIDs and single pixel TESs and discuss the experimental characterisation of the electrical and thermal properties of the absorber and TES, as well as the device as a whole. From these results, and in conjunction with the modelling algorithms of Chapter 4, we discuss optimisations of the current devices which will lead to improved performance. Chapter 7 reports on the further experimental characterisation of the devices, specifically, X-ray testing of the DROID and single pixel TES. Finally, in Chapter 8, we summarise the theoretical modelling and experimental results discussed in the previous Chapters and outline the future research aims based on these findings.

Chapter 2: Transition Edge Sensor Theory

Chapter 2 is a comprehensive review of the fundamental physics of microcalorimeter detectors. Including, semiconductor thermistors, Transition Edge Sensors (TESs) and Metallic Magnetic Calorimeters (MMCs), all under development as X-ray detectors by numerous groups world-wide. Particular attention is paid to the details of TES detectors operating under electro-thermal feedback conditions.

2.1. Microcalorimeters as X-ray Detectors

2.1.1. Calorimeter Basics

The capability of cryogenic calorimeters (operating at temperatures < 100 mK) to sensitively measure small temperature changes has been explored for many years. An early example is found in nuclear physics, where a 1 gram iron ammonium alum calorimeter, cooled to 50 mK, was used to measure the energy released in the radioactive decay of radon [Simon, 1935]. The measurement of energy from single particle or photon events was not however realised until the much more recently [Moseley et al., 1984; Fiorini and Niinikoski, 1984].

The development of microcalorimeters is not only important to the astrophysics community, providing high resolution spectroscopy and imaging from the microwave to γ -ray band of the electromagnetic spectrum but, potentially, in many other fields such as non-destructive materials analysis [Hollerith et al., 2004; Wollman et al., 1997] and quantum information [Nam et al., 2004]. Microcalorimeters are also being developed for energetic particle detection; α -particles [Frank et al., 1994] and neutrons [Niedermayr et al, 2004] for example.

A calorimeter in its simplest form consists of an absorber, with heat capacity C, in thermal contact with a thermometer. The calorimeter is thermally linked via a conductance G, to a heat sink or cold bath, which is at a reference temperature T_b . The deposition of energy E into the system by photon or

particle interactions will result in a temperature rise in the absorber ΔT , which is measured by the thermometer. Figure 2.1 shows a schematic of the simple calorimeter concept.



Figure 2.1. Left – schematic diagram of a microcalorimeter. An absorber with heat capacity C is coupled to a heat sink via a thermal conductance G. Right – the time evolution of the temperature profile measured with the thermometer, after the deposition of energy E into the absorber.

The behaviour of the system can be represented by the power balance equation:

$$C\frac{dT(t)}{dt} = P_{\gamma} - P_{link} , \qquad 2.1$$

where P_{link} is the heat loss to the cold bath and P_{γ} is the power input from the photon. For temperatures close to T_{b} , P_{link} is approximated by:

$$P_{link} = G(T(t) - T_b).$$

Substituting for P_{link} in Equation 2.1 and solving for T(t) produces:

$$T(t) = T_b + \Delta T(t), \qquad 2.3a$$

where,

$$\Delta T(t) = \Delta T_0 \exp\left(\frac{-t}{\tau_0}\right).$$
 2.3b

In Equation 2.3b, $\tau_0 = C/G$ is the time constant governing the exponential decay of the initial temperature rise ΔT_0 , back to T_b . Ideally, the energy is deposited instantaneously leading to an initial temperature rise $\Delta T_0 = E/C$. Consequently for a known *C*, the measurement of ΔT gives a direct measurement of the energy deposited in the absorber. By measuring the energy of many individual events it is then possible to obtain a spectrum of the source of radiation. Devices that measure the energy of individual particles or photons are called micro or quantum-calorimeters, whereas those that measure the power from a flux of many photons are usually referred to as bolometers. The focus of this thesis is the microcalorimeter type of detector used for single photon counting applications.

In any calorimeter there exists a background noise level caused by random thermodynamic fluctuations. From statistical mechanics it can be shown that the root-mean squared (rms) magnitude of these energy fluctuations ΔE_{rms} in the calorimeter at temperature *T* is given by [Mandl, 1971]:

$$\Delta E_{rms} = \sqrt{k_b T^2 C} , \qquad 2.4$$

where k_b is Boltzmann's constant. These fluctuations are independent of both the energy deposited E, and the thermal conductance G (though the frequency spectrum of these fluctuations does depend on G). The performance of the detector can therefore be evaluated knowing only the heat capacity of the system C and the operating temperature T. This leads to the basic requirements for a microcalorimeter to have a low heat capacity and low operating temperature. We note that the heat capacity is also a function of temperature C(T) and will depend on the materials used for the thermometer and absorber.

For a complete description of the energy resolution of these detectors we must not only include these thermodynamic fluctuations (or phonon noise), but also additional sources of noise (generated by the thermometer, for example) and the responsivity of the thermometer itself. The sources of noise that will limit the detector performance will be examined in detail in Section 2.2.3. Moseley et al. (1984) show that by considering these additional factors, an expression can be derived which differs from Equation 2.4 only by a dimensionless numerical factor ξ , which depends primarily on the design and type of thermometer used:

$$\Delta E_{rms} = \xi \sqrt{k_b T^2 C} \,. \tag{2.5}$$

Microcalorimeter thermometers that are based on a material whose resistance is a function of temperature, are known as thermistors. Thermistors generally come in two forms: (i) semiconductor thermistors [Moseley et al., 1984] and (ii) Transition Edge Sensors (TESs) [Irwin, 1995a; Irwin et al., 1996] sometimes referred to as Superconducting Phase Thermometers (SPTs) [Seidel et al., 1990]. A second type of calorimeter has more recently been identified; these are known as Metallic Magnetic

Calorimeters (MMCs) [Fleischmann et al., 2003]. In this case, the thermometer is a material whose magnetisation is a function of temperature. In Sections 2.1.2-2.1.4, we review each type.

2.1.2. Semiconductor Thermistors

Early microcalorimeter development used doped semiconductor thermistors made from ion-implanted silicon and neutron transmutation doped (NTD) germanium. These thermistors have been successfully demonstrated as microcalorimeters for both X-ray [Moseley et al., 1984; McCammon et al., 1993] and particle detection [Fiorini and Niinikoski, 1984; Wang et al., 1989].

The conduction of a semiconductor thermistor, in the temperature range 50 mK to 4 K, is well described by the variable range hopping (VRH) mechanism [Efros and Shklovskii, 1975], which leads to an electric resistance R, which is strongly dependent upon temperature T:

$$R = R_0 \exp\left(\sqrt{\frac{T_0}{T}}\right),$$
 2.6

where R_0 and T_0 are constants depending on the thermistor doping, although R_0 also depends on the thermistor dimensionality. The sensitivity of the thermistor is characterised by a dimensionless constant α , often referred to as the logarithmic sensitivity of the thermometer:

$$\alpha = \frac{T}{R} \frac{dR}{dT} = \frac{d\ln R}{d\ln T}.$$
 2.7

For semiconductor thermistors, the resistance increases as the temperature decreases, consequently α is negative and typically takes values between -3 and -8 [McCammon et al., 1993]. Semiconductor thermistors are current biased so that when a photon or particle interacts with the absorber, the resulting resistance drop causes a negative voltage pulse. Semiconductor thermistors typically have resistances of ~ 10 M\Omega and are therefore well matched to the impedance of silicon junction field effect transistors (JFETs), which are usually operated at 77 K. Since thermistors are relatively insensitive (small α), in order to minimise ΔE , these early microcalorimeters incorporated insulating or superconducting absorbers with a low intrinsic heat capacity compared to a metal (see Section 6.1). For semiconductor thermistors the numerical factor ξ in Equation 2.5 has a value ranging between about 1.5 and 3.0, scaling as α^{-1} at lower sensitivities and $\alpha^{-1/2}$ at larger sensitivities [McCammon et al., 1993], thus making energy resolutions of a few eV possible for an optimally designed device. Full width at half maximum (FWHM) energy resolution values of 7.3 eV for 5.9 keV X-rays have been experimentally achieved [McCammon et al., 1991] for a doped Si thermometer with HgTe absorber (in this case a non-linear R(T) function degraded the resolution above the expected value of 4.5 eV)

and more recently, 3.1 eV at 5.9 keV for a NTD Ge device with Sn absorber [Silver et al., 2005]. These results are comparable to the resolution achieved with the latest TES detectors discussed in the next Section. Further improvements in thermistor resolution have proved difficult to achieve, largely due to the fact that further reductions in the heat capacity for already small absorbers are difficult but also as a result of the statistical variations in the thermalisation of the X-ray energy in non-metallic absorbers (see Section 6.1).

The major disadvantage with the semiconductor thermistors is that they have an intrinsically slow response time (with decay time constants of $\tau_0 \ge 1$ ms [McCammon et al., 1993] compared to ~ 100 μ s for TES detectors), which limits the count rate capability. Despite this disadvantage, flight semiconductor thermistor arrays have been developed for X-ray astronomy. The first astrophysical observations using a microcalorimeter detector were made by the X-ray Quantum Calorimeter (XQC) sounding rocket experiment (developed by Wisconsin and NASA Goddard Space Flight Centre), which had two successful flights in 1996 and 1999 [Porter et al., 2000a]. The XQC used a 36-pixel ion implanted silicon thermistor array, with HgTe absorbers, for measuring the soft X-ray background from 0.02-1 keV. The QXC detector achieved a FWHM energy resolution of ~ 9 eV at 1.1 keV [Porter et al., 2000b]. Following on from these successful experiments was the Astro-E mission [Kelly et al., 2000], designed to put a 32 × 32 microcalorimeter pixel array into Earth orbit. Unfortunately, this observatory failed to achieve the correct orbit on launch in 2000 and the mission failed. This array was capable of FWHM energy resolutions of $\sim 12 \text{ eV}$ at 5.9 keV and $\sim 8 \text{ eV}$ below 1 keV, with a decay time constant of $\tau \approx 6$ ms [Stahle et al., 1999]. The next generation XRS has been developed for Astro-E2 (renamed Suzuka after launch in 2005). The results obtained with this new XRS show a 5.3-6.5 eV FWHM energy independent resolution with decay times of $\tau \approx 3$ ms [Stahle et al., 2004]. This new instrument demonstrates a factor of ~ 2 improvement in both resolution and count rate when compared to the original XRS. Sadly, after a successful launch, the XRS operations on Astro-E2 were ended after the cryogen reserves for the cooler unexpected ran out after only a few weeks. For further details on semiconductor thermistors see McCammon (2005).

2.1.3. Transition Edge Sensors

Over the last decade microcalorimeter development has focused more on thermistors based on superconducting thin films. A Transition Edge Sensor (TES) consists of a thin (~ 100 nm) metallic film with a superconducting-to-normal (S-N) transition temperature T_c , typically of the order 0.1 K. TES detectors have a normal state resistance $R_n \leq 1 \Omega$, making them less susceptible to microphonic pickup than the high-impedance semiconductor thermistors [Wollman et al., 1997]. A TES is operated on the sensitive transition edge between the S-N states where small changes in temperature ΔT , result in large resistance changes ΔR . The TES is usually direct current (DC) voltage biased, although alternating current (AC) biasing is also possible [Cunningham et al., 2002]. An increase in resistance due to the deposition of energy from an X-ray photon causes a negative current pulse. Unlike semiconductor thermistors, TESs are not well impedance matched to silicon JFETs, consequently, an alternative read-out in the form of an inductively coupled Superconducting Quantum Interference Device (SQUID) [Gallop, 1991] is commonly used. The details of SQUID performance and operation will be discussed in Chapter 5, but in general, SQUIDs have the advantages of being able to operate at mK temperatures and producing less noise than JFETs. The TES and absorber are thermally isolated on a thin silicon nitride membrane (\sim 500 nm), which provides a weak thermal link (*G*) between the TES and the cold bath. Figure 2.2 shows a schematic diagram of a TES based microcalorimeter.



Figure 2.2. Schematic diagram of a TES microcalorimeter. The detector consists of a TES and absorber thermally coupled to a Si substrate via a SiN membrane.

The operation of a TES incorporates the effect of negative electro-thermal feedback (ETF) [Irwin, 1995a]. In this régime, the applied voltage bias V_0 , sets up a current through the film, which is dependent upon V_0 and the TES resistance R(T). The voltage bias is achieved using a small shunt/load resistor, in parallel with the TES. In order to achieve optimum voltage bias the load resistance has a much smaller value than the operating resistance of the TES R_0 . Since the bias is constant across the TES; as the detector cools through the S-N transition, R(T) decreases and the bias current increases. The applied bias gives rise to Joule heating in the film $P_0 = V_0^2/R(T)$, which increases as the TES resistance drops. This heating effect maintains the TES at a higher temperature than the cold bath $(T_b$ < 50 mK). A stable equilibrium is reached when the Joule heating in the film is matched by the heat loss through the weak thermal link to the bath P_{link} . By careful adjustment of V_0 , the TES can be maintained in the narrow region between the superconducting and normal states. This process is known as negative electro-thermal feedback (ETF). An ETF-TES is essentially a self-biasing device such that fluctuations in temperature are countered by opposing changes in P_0 , this makes it possible to hold a very stable equilibrium bias point in the narrow S-N transition region. A sudden increase in the TES temperature caused by the absorption of a photon, will result in an increase in R(T) and hence a drop in P_0 in the film. Hence the Joule heating drops and the TES cools back to the equilibrium bias

point at a rate faster than if P_0 were maintained a constant. The effect of the negative ETF is thus to speed up the removal of heat, therefore speeding up the response time of the detector by a reduction in the decay time below the intrinsic time constant ($\tau_0 = C/G$). Typically this reduction is by an order of magnitude – see Section 2.2.1.

The transition edge is typically only a few mK wide. Consequently, temperature excursions due to photon absorption give rise to large increases in ΔR . Figure 2.3 shows a typical resistance versus temperature (*R*-*T*) transition curve for an iridium TES resistor. As with semiconductor thermistors, the sensitivity of *R*(*T*) in the transition is described by the dimensionless parameter α , which for a TES can be as high as 1000, though more typically under operating conditions $\alpha \sim 100$. This sensitivity is typically over an order of magnitude higher than for semiconductor thermistors and means that despite having higher specific heats, metal absorbers can be used to improve photon thermalisation and hence energy resolution. For TES detectors the parameter ξ in Equation 2.5 is $\sim 2.5 \alpha^{-1/2}$ [Irwin et al., 1996], therefore such high sensitivities mean that ξ can be as small as 0.1, making better energy resolution theoretically possible compared to semiconductor thermistors.



Figure 2.3. Resistance vs. temperature plot for the superconducting to normal transition of an iridium TES resistor. The data is taken from Trowell (2001). The inset shows a typical current pulse as seen on the output of the detector.

Theoretically, TES based detectors offer better energy resolution and higher count rate capabilities than their semiconductor counterparts; consequently there is great interest in developing these detectors as imaging spectrometers in X-ray astronomy. Single pixel TES detectors are under development by a number of groups around the world with promising results achieved so far (2.4 eV at 5.89 keV [Ullom et al., 2005] and 2.0 eV at 1.49 keV [Wollman et al., 2000]). Great challenges still

lie ahead in developing large format TES arrays for missions such as XEUS [Bruijn et al., 2004] or Constellation-X [Lindeman at al., 2004a]. Imaging with TESs will be discussed further in Chapter 3. Though the performance of pixel detectors is impressive, it is still poorer than that predicted by the theory. This under performance in energy resolution is due to non-ideal effects [Galeazzi, 2004a] and unexplained sources of 'excess' noise internal to the TES [Takei et al., 2004; Lindeman et al., 2004b; Staguhn et al., 2004; Bergmann Tiest et al., 2004]. There is currently a great deal of effort being invested by the TES community in characterising this excess noise and developing methods of reducing it in order to further improve resolution [Galeazzi et al., 2004b; Ullom et al., 2004; Fraser, 2004; Seidel and Beloborodov, 2004]. Further details of TES operation is described in Section 2.2

2.1.4. Metallic Magnetic Calorimeters

A Metallic Magnetic Calorimeter (MMC) consists of an X-ray absorber thermally coupled to a material whose magnetisation obeys a 1/T Curie law [Hook and Hall, 1999] dependence, at temperatures < 0.1 K. The absorption of energy δE in a detector with volume V and heat capacity C causes a change in magnetic moment δm , given by:

$$\delta m = V \frac{\partial M}{\partial T} \frac{\delta E}{C} \,. \tag{2.8}$$

Changes in magnetic flux caused by energy deposition are measured using a SQUID magnetometer situated in close proximity to the sample. The sensor is magnetised by a small B-field, typically of the order a few mT, in the plane perpendicular to the SQUID pickup coil. A material with large $\partial M/\partial T$ is required to maximise sensitivity. The magnetisation in MMCs comes from a dilute concentration of paramagnetic ions in a metallic host. This dilute concentration of randomly distributed ions in a host material is essential to limit the spin-spin interaction between ions, which result in an excess heat capacity and a decrease in magnetisation. For an MMC, the theoretical FWHM energy resolution is again described by Equation 2.5 with the parameter $\xi \sim 2\sqrt{2}(\tau_0 / \tau_1)^{1/4}$, which is typically < 1 [Fleischmann et al., 2003]. This ultimate resolution is limited by fluctuations in energy exchange between the spins and the absorber. As well as requiring a low operating temperature *T* and a small heat capacity *C*, the resolution depends on the system time constants τ_0 and τ_1 (unlike a thermistor). Here $\tau_0 (\sim 10^{-6} \text{ s})$ is the thermal equilibrium time constant between the spins and the absorber and τ_1 (~ 10^{-4} s) is the thermal time constant between the spins and the absorber of $\tau_0 \sim 10^{-4}$ s) is the thermal time constant between the spins and the absorber of $\tau_1 \sim 10^{-4} \text{ s}$ single to the spin sum of $\tau_1 \sim 0^{-4} \text{ c}$ so that for TESs (~ eV for X-ray band), with decay times of ~ 100 µs, but is superior for longer thermal decay times. Thus, there exists a trade-off between resolution and count rate.

The MMC can be deposited directly on a silicon substrate without the need for thermal isolation on thin silicon nitride membranes, as is required for TESs operated under electrothermal feedback conditions. Furthermore, MMCs generate a change in magnetic flux on photon absorption so can be directly flux coupled (or transformer coupled) to a SQUID, with no need for any electrical bias circuitry, as is required for TES detectors. Consequently, there are no large heat loads on the detector plane, which in turn greatly reduces the thermal constraints that makes TES array development such a challenge (see Chapter 3).

The choice of host material and embedded ion is driven by technical constraints such as ease of fabrication and long term chemical stability, in addition to the magnetic properties. The existence of – Ruderman Kittel Kasuya Yosida (RKKY) [Ruderman and Kittel, 1954] – exchange interactions between spins will have a detrimental effect on MMC performance by reducing $\partial M/\partial T$ and hence, the device sensitivity [Enss et al., 2000]. The use of rare earth elements in the 4f series (Er, Dy, Ho for example) is most suited in this regard due to their weak interaction with the conduction electrons in a metallic host. MMC development has so far concentrated entirely on Au doped with Er ions [Fleischmann et al., 2000], largely because the low temperature properties are well documented. Au is ideal as a host material from the point of the view of ease of fabrication and rapid thermalisation but does suffer from a high electronic heat capacity, which in turn limits resolution.

Early results in single pixel metallic magnetic calorimeters (MMCs) have demonstrated an energy resolution of ~ 3.4 eV FWHM [Fleischmann et al., 2004] for 5.9 keV X-rays, comparable to the latest performance of thermistors. Yet further improvements in resolution are predicted for fully optimised devices, MMCs therefore offer an exciting prospect for focal plane X-ray detectors. Despite the rapid emergence of MMCs, these cryogenic detectors are globally still in their infancy. To date, MMC research has almost entirely been based around a collaboration between Brown University (USA) and the University of Heidelberg (Germany) [Bandler, 2004], though further groups in both Europe and America are now perusing this alternative technology.

At the University of Leicester we have recently started, in collaboration with the Condensed Matter Physics group, developmental research into MMCs based around alternative paramagnetic systems (Au:Dy, Bi:Dy for example), and as such have measured the high temperature high field 1/T Curie dependence of a Au:Dy (1000 ppm) sample which conforms to the theoretical predictions (see Figure 2.4).



Figure 2.4. First magnetisation data (taken by Dr. Steve Baker) of an Au:Dy sample versus inverse temperature at a magnetic field of 1 T, for temperatures between 2 K and 50 K. The sample volume dimensions are 1 cm \times 1 cm \times 4.1 μ m, and the Dy concentration is 1000 ppm. The linear region at high temperature clearly shows the 1/T Curie dependence which is measured to be as high as ~ 11200, compared to a theoretical value of ~ 13900.

2.2. Detailed Examination of TES Theory

Now that the principles of microcalorimeter operation have been outlined, we are able to take a closer look at the details of TES theory.

2.2.1. The Effect of Negative ETF on Decay Time and Energy Integral

In a TES detector, the heat loss through the weak thermal link P_{link} , can be expressed by the power law:

$$P_{link} = K \left(T_0^n - T_b^n \right), \qquad 2.9$$

where *n* is a dimensionless constant (or thermal exponent), which depends on the dominant thermal transport mechanism between the TES and the cold bath, typically taking a value between 3 and 6 ($n \approx$ 3 indicates SiN dominated transport, $n \approx 4$ indicates Kapitza boundary resistance [Swartz and Pohl,

1989], $n \approx 5-6$ indicates electron-phonon coupling [Wellstood F. et al., 1994]). The constant K is geometry and material dependent and has units of W K⁻ⁿ. The temperature at the operating point is T_0 and T_b is the bath temperature. The overall thermal conductance G of the weak link is then defined as:

$$G = \frac{dP_{link}}{dT} = nKT^{n-1} W K^{-1}.$$
 2.10

G is typically of the order of 10^2 pW K⁻¹. In Chapter 6 we experimentally investigate the thermal properties of our current TES devices and numerically estimate the key parameters *n*, *K* and *G*.

Including the Joule heating terms and the power law expression for the heat loss through the membrane, the power balance equation describing the ETF-TES (compared to Equation 2.1 for the simple calorimeter model) is:

$$C\frac{dT}{dt} = \frac{V^2}{R(T)} - K(T_0^n - T_b^n) + P_{\gamma}.$$
 2.11

This can be linearised for small changes in temperature ΔT around the equilibrium operating point by using a Taylor expansion:

$$\frac{d\Delta T}{dt} = -\left(\frac{G}{C} + \frac{P_0 \alpha}{T_0 C}\right) \Delta T + P_{\gamma}, \qquad 2.12$$

where P_0 is the Joule power at the equilibrium bias point. The solution to Equation 2.12 is given by the expression:

$$\Delta T(t) = \frac{E}{C} \exp\left(\frac{-t}{\tau_{eff}}\right),$$
2.13

the same as Equation 2.3b for the simple model with no ETF, but now we have an effective time constant τ_{eff} given by:

$$\tau_{eff} = \frac{\tau_0}{1 + \frac{P_0 \alpha}{T_0 G}} \,. \tag{2.14}$$

The dimensionless parameter $P_0 \alpha / T_0 G$ is often referred to as the 'loop gain' L_0 and by using Equations 2.9 and 2.10 can be written as:

$$L_0 = \frac{P_0 \alpha}{T_0 G} = \frac{\alpha}{n} \phi , \qquad 2.15$$

where ϕ is a factor which depends on the temperature difference between the TES and the cold bath:

$$\phi = 1 - \left(\frac{T_b}{T_0}\right)^n.$$
 2.16

and is approximately equal to 1 for $T_b \ll T_0$. The effect of negative ETF is to reduce the pulse decay time by the factor $1+L_0$. If the TES was current biased then the denominator in Equation 2.14 would be $1-L_0$ and we would be operating under positive ETF conditions. As $T_b \rightarrow T_0$ then $\tau_{eff} \rightarrow \tau_0$ and there is no ETF. Alternatively, in the extreme ETF regime $L_0 \gg 1$ (that is $T_b \ll T_0$ with $\alpha/n \gg 1$) and Equation 2.14 simplifies to:

$$\tau_{eff} \approx \frac{n\tau_0}{\alpha}.$$

For typical values of $\alpha = 90$ and n = 3, $\tau_{eff} \sim \frac{\tau_0}{30}$.

This calculation shows that under extreme ETF conditions, an order of magnitude improvement in response time over the intrinsic thermal relaxation time is easily achievable. This provides a major advantage over the semiconductor thermistors where the decay time is limited to τ_0 .

The concept of the ETF-TES can be further illustrated if we consider an effective thermal conductance G_{eff} , which is responsible for heat removal from the system. G_{eff} is then the parallel combination of the physical thermal link to the cold bath G and the virtual thermal conductance due to the ETF G_{ETF} :

$$G_{eff} = G + G_{ETF} \,. \tag{2.18}$$

The effective time constant of the system τ_{eff} can then be expressed as:

$$\tau_{eff} = \frac{C}{G_{eff}} = \frac{C}{G + G_{ETF}}.$$
 2.19

Equating this expression to Equation 2.14 we find that:

$$G_{ETF} = L_0 G = \frac{P_0 \alpha}{T_0}$$
. 2.20

Similarly, the ETF time constant is:

$$\tau_{ETF} = L_0 \tau_0 \,. \tag{2.21}$$

Thus if $L_0 >> 1$ we have the extreme ETF régime and $G_{ETF} >> G$. Conversely, as $L_0 \rightarrow 0$ the effect of the ETF rapidly reduces and G_{ETF} disappears. It is now clear that under extreme ETF conditions the removal of heat generated by an X-ray photon is dominated by G_{ETF} and not G. Consequently the heat flow to the substrate remains largely constant and the energy of the absorbed photon is removed by the change in Joule power. The change in Joule power ΔP , or the integral of the current pulse over time, will therefore give an estimate of the energy of the absorbed photon:

$$E_{joule} = \int_{0}^{\infty} \Delta P dt = V_0 \int_{0}^{\infty} \Delta I dt .$$
 2.22

The current in the TES can be found from [Irwin, 1995b]:

$$\Delta I = \frac{V_0}{\Delta R} = -\frac{V_0}{R^2} \frac{dR}{dT} \Delta T = -\frac{I_0 \alpha}{T_0} \Delta T = -\frac{I_0 \alpha}{T_0} \frac{E}{C} \exp\left(\frac{-t}{\tau_{eff}}\right).$$
 2.23

Substituting for ΔI into Equation 2.22 and integrating, gives the energy removed by the Joule heating:

$$E_{joule} = -V_0 \tau_{eff} \frac{I_0 \alpha}{T_0} \frac{E}{C} = \frac{1}{1 + \frac{1}{L_0}} E.$$
 2.24

Consequently, as shown in Figure 2.5, $E_{joule}/E \rightarrow 1$ as $L_0 \rightarrow \infty$, it is only under extreme ETF conditions that the energy is removed entirely by the change in Joule heating. For a typical TES, with $\alpha = 100$ and $T_0 = 100$ mK we would expect $E_{joule}/E = 0.96$.



Figure 2.5. Showing the ratio of the energy from the change in Joule power to the actual photon energy, E_{joule}/E , as a function of the loop gain L_0 .

2.2.2. Effect of Current Dependence and Non-Perfect Voltage Bias

The effect of non-perfect voltage bias and the fact that the resistance of the TES depends on the current flowing through it must all be taken into account in a complete description of the TES response [Feliciano-Figueroa, 2001]. The analysis in Section 2.2.1 is simplified by the assumptions that the voltage bias is perfect and that the TES resistance depends only on temperature.

The quality of the TES voltage bias will affect the detector response. Voltage bias is achieved using a small shunt/load resistor R_l in parallel with the TES. Perfect voltage bias conditions only occur when $R_l \ll R_0$, conversely if $R_l \gg R_0$, the detector will be current biased. The quality of the bias can be parameterised using, $(R_0 - R_l)/(R_0 + R_l)$, which is equal to 1 for perfect voltage bias and -1 for perfect current bias.



Figure 2.6. The venin [Hamilton, 2003] equivalent electrical bias circuit coupled to the TES. V_{nl} is the Johnson noise from the load resistor, V_{nl} is the Johnson noise from the TES, P_n is the phonon noise due to the weak link and L_{in} is the inductance of the SQUID input coil.

The resistance of the TES is not only a function of temperature but also current (and in fact magnetic field), R(I,T) [Lindeman et al., 2004c]. Therefore the TES resistance can no longer be considered in terms of a simple *R*-*T* transition; it must be considered as a three dimensional *R*-*I*-*T* surface. This surface is characterised by two dimensionless parameters α_T and α_I , which are defined as:

$$\alpha_T = \frac{T}{R} \frac{\partial R}{\partial t} = \frac{\partial \ln R}{\partial \ln T},$$
 2.25a

$$\alpha_I = \frac{I}{R} \frac{\partial R}{\partial I} = \frac{\partial \ln R}{\partial \ln I}.$$
 2.25b

These are the gradients of the *R-I-T* surface, with α_T evaluated for constant current and α_I evaluated for constant temperature. When the detector is under operational bias conditions, changes in temperature will also induce changes in current. The resulting changes in resistance ΔR from small changes in both current and temperature, ΔI and ΔT respectively, will be:

$$\Delta R = \frac{R}{I} \alpha_T \Delta T + \frac{R}{T} \alpha_I \Delta I .$$
 2.26

The shape of the S-N transition under bias conditions is usually determined by measuring the TES current as a function of voltage $I_{TES}(V)$ throughout the transition (see Chapter 7 for experimental iridium TES *I-V* curves), from which, it is possible to estimate the *R-T* curve and hence, α . Sweeping the bias current through the transition in this way will result in movement across the *R-I-T* surface.

The α value from the *R*-*T* curve determined in this way will actually be an effective α incorporating both α_T and α_I . Under perfect voltage bias conditions we have [Takei et al., 2004]:

$$\alpha = \frac{\alpha_T}{1 + \alpha_I}, \qquad 2.27$$

which can be substituted directly into Equation 2.15 for L_0 . Typically $0 < \alpha_I < 1$ [Lindeman et al., 2004c] which results in a measured α smaller than α_T .

To investigate the combined effect of both a current dependence and non-perfect bias, the system response must be re-derived. From Figure 2.6, the electrical response of the TES, including both R(T,I) and the load resistor R_I , is represented by:

$$L_{in}\frac{dI}{dt} = V_b - I(t)R(T,I) - I(t)R_l .$$
 2.28

Equations 2.28 and 2.19 (for the thermal system) can again be linearised and by using the definitions for *G*, α_T and α_I from Equations 2.10, 2.25a and 2.25b respectively:

$$L_{in}\frac{d\Delta I}{dt} = -[R_I + R_0(1 + \alpha_I)]\Delta I - \frac{V_0\alpha_T}{T_0}\Delta T.$$
2.29

For the thermal power balance we again linearise Equation 2.11 but with the assumption that resistance is now a function of both temperature and current. We then have:

$$C\frac{d\Delta T}{dt} = V_0 (2 + \alpha_I) \Delta I + \left[\frac{P_0 \alpha_T}{T_0} - G \right] \Delta T + P_{\gamma}.$$
 2.30

Initially setting L = 0, and solving Equations 2.29 and 2.30 for ΔI leads to [Feliciano-Figueroa, 2001]:

$$\Delta I = \frac{\alpha_T}{(1+\alpha_I)R_0 + R_l} \frac{V_0}{T_0} \frac{E}{C} \exp\left(\frac{-t}{\tau_{eff}}\right),$$
2.31

with an effective time constant,

$$\tau_{eff} = \frac{\tau_0}{1 + L_0 \beta} \,. \tag{2.32}$$

 L_0 has the same meaning as in Equation 2.15 except that α is replaced with α_T and the parameter β includes the 'quality' of the voltage bias and the current dependence:

$$\beta = \frac{R_0 - R_l}{(1 + \alpha_l)R_0 + R_l}.$$
 2.33

The parameter β therefore varies from -1 to +1 depending on the values of α_l and R_l . We see that a non-zero α_l and R_l will directly modify the ETF, reducing the pulse height and increasing the effective decay time. The ETF thermal conductance is therefore also modified as follows:

$$G_{ETF} = \beta L_0 G = \frac{P_0 \alpha_T}{T_0} \frac{R_0 - R_l}{(1 + \alpha_I)R_0 + R_l} .$$
 2.34

and will be reduced for non-zero values of α_l and R_l . To maintain extreme negative ETF operating conditions we require $R_l \ll R_0$ and typically $R_l \ll R_0/10 \ll R_n/100$ where R_n is the normal state resistance of the TES. From Equation 2.31 it is clear that for a stable detector response we require a positive τ_{eff} , this in turn means that $G_{ETF}/G = \beta L_0 > -1$. This implies that even with $R_l > R_0$ and positive ETF (i.e. $G_{ETF} \ll 0$), the response can be stable (so long as $\beta L_0 > -1$).

The energy of the pulse can again be estimated by integrating the pulse shape. However, because of the non-perfect voltage bias ($V_0 \neq constant$), this measured energy will not be the same as the actual change in Joule power ($E_{joule} \neq E_{meas} = V_0 \int \Delta I \, dt$) and hence the true energy of the event. Using the same method used to derive Equation 2.24, assuming V_0 is a constant, we have for the integrated energy:

$$E_{meas} = \left(\frac{R}{R - R_l}\right) \frac{1}{1 + \frac{1}{\beta L_0}} E .$$
 2.35

The actual energy removed from the TES due to the change in Joule power will in fact be [Feliciano-Figueroa, 2001]:

$$E_{joule} = \int_{0}^{\infty} \Delta P dt = \left(V_0 - I R_l \right) \int_{0}^{\infty} \Delta I dt = \frac{1}{1 + \frac{1}{\beta L_0}} E,$$
 2.36

which is the same as Equation 2.24 except for the modified loop gain βL_0 .



Figure 2.7. The energy estimated from integrating the pulse shape using the incorrect assumption that $V_0 = \text{constant}$ and by considering the actual change in Joule power. The data is plotted as a function of β , with $\alpha_I = 0$.

Figure 2.7 shows a plot of E_{meas} and E_{joule} (as a fraction of the incident energy *E*) as a function of β in the range 0 to 1. The parameter β is varied by changing R_l for a fixed R_0 and α_l , with $T_b = 0$. When $\beta = 1$, E_{meas} and E_{joule} are both equal to the photon energy *E* as we are in the strong ETF limit with perfect bias. As β is decreased, E_{meas} and E_{joule} diverge; the decreasing quality of voltage bias reduces the effect of the ETF, hence, the energy removed by the change in Joule power decreases. However, because the voltage across the TES is not constant, a 'softer' voltage bias means that use of the invalid assumption of $V_0 = constant$, results in an increase in the estimated energy.

2.2.3. Noise and Energy Resolution

In an ideal TES detector the only sources of noise will be Johnson noise in the TES and phonon noise (sometimes referred to as thermal fluctuation noise or TFN) caused by the thermal link to the cold bath.

Johnson (or Nyquist) noise V_{nt} , expressed in units of V Hz^{-1/2} [Johnson, 1928; Nyquist, 1928], is internal to the TES and arises due to the random thermal motion of charge carriers. This random motion results in a white voltage noise with a spectral density given by:

$$V_{nt} = \sqrt{4k_b R_0 T_0} , \qquad 2.37$$

and is dependent on the resistor value R_0 and the temperature of the resistor T_0 ; k_b is Boltzmann's constant.

Phonon noise P_n (W Hz^{-1/2}) originates from thermodynamic fluctuations across the weak thermal link *G* between the TES and the cold bath. The phonon noise will have a power spectral density given by [Mather, 1982]:

$$P_n = \sqrt{4\gamma k_b T_0^2 G} , \qquad 2.38$$

where the factor $\gamma = n/(2n+1) \sim 0.4$ -0.5 is a dimensionless constant which takes into account a temperature gradient across the weak link. If the link is isothermal, then no temperature gradient exits and $\gamma = 1$.

As well as these unavoidable sources of noise there may exist additional noise from the read-out circuitry and the SQUID and amplifier chain, though with care these can be eliminated. The load resistor will also contribute a Johnson noise source with magnitude:

$$V_{nl} = \sqrt{4k_b R_l T_l} , \qquad 2.39$$

where R_l and T_l are the resistance and temperature of the load resistor respectively. Generally speaking, for an optimum voltage bias $R_l \ll R_0$, therefore $V_{nl} \ll V_{nt}$ and V_{nl} is negligible.

An important effect of the ETF is that it modifies the frequency response of the signal and the noise. All sources of noise internal to the TES (i.e. the TES Johnson noise) will be suppressed for frequencies $f < (2\pi\tau_{eff})^{-1}$ (an effect first reported by Mather (1982)). The ETF modifies both the signal and the noise in the same way; therefore the signal-to-noise ratio (*SNR*) and hence the overall detector resolution remains unaffected compared to a TES without ETF. In the simplest case, with $R_l = 0$ and $\alpha_l = 0$, the noise spectral densities on the detector input for an ETF-TES can be written as:

$$i_{johnson} = \frac{1}{(1+L_0)} \sqrt{\frac{1+\omega^2 \tau_0^2}{1+\omega^2 \tau_{eff}^2}} \frac{V_n}{R_0},$$
 2.40

$$i_{phonon} = \frac{L_0}{(1+L_0)} \sqrt{\frac{1}{1+\omega^2 \tau_{eff}^2}} \frac{P_n}{V_0},$$
 2.41

for the TES Johnson noise and the phonon noise respectively, which, in the strong ETF limit are equivalent those derived by Irwin (1995b). The angular frequency is given by $\omega = 2\pi f$. Figure 2.8 show two plots of the calculated Johnson and phonon noise as well as the signal in a typical ETF-TES. The two plots are for $\alpha = 1$ and $\alpha = 100$ respectively. It is clear from Equation 2.40 that the TES Johnson noise is suppressed by the factor $1/(1+L_0)$ in the low frequency limit ($\omega^2 \rightarrow 0$) and tends to the white noise limit as $\omega^2 \to \infty$. Increasing α by 100 (and therefore L_0) decreases the low frequency limit of the detector Johnson noise. The phonon noise is white up to a corner frequency of $f_{eff} = (2\pi\tau_{eff})^{-1}$ corresponding to the -3dB point, after which the temperature fluctuations are damped. Note that the frequency response of the signal is identical to that of the phonon noise. In the zero frequency limit, for the case where $L_0 >> 1$, the phonon noise reduces to, $i_{phonon} = \sqrt{4nk_bT_0/R_0}$, thus differing from the white Johnson noise level by the factor \sqrt{n} . In contrast to a non-ETF-TES, increasing α has the effect of pushing the f_{eff} 'knee' out to higher frequency by decreasing τ_{eff} as described in Equation 2.14. For low values of α the Johnson noise will dominate at all frequencies whereas at high values of α the phonon noise dominates for $f < f_{eff}$ and the Johnson noise dominates for $f > f_{eff}$. As for a non-ETF-TES, increasing α will increase the pulse amplitude, consequently the signal crosses over the Johnson noise at ever increasing frequencies and the total SNR increases. The SNR affects the detector energy resolution so an increase in SNR will improve the energy resolution of the TES, as long as the bandwidth ΔB , is increased proportionally. This implies that large α devices are desirable for a high energy resolution performance. There is a limit on how much the frequency can be increased. This limit is determined by the thermalisation and diffusion times in the TES/absorber. In the absence of any circuit inductance the pulse rise time will be limited by the thermalisation time. No information will exist above the thermalisation frequency of the absorber. Therefore an upper limit is set on the frequency response of the signal and hence the usable bandwidth. Care must be taken to insure that thermalisation occurs at higher frequencies than f_{eff} , otherwise a second pole in signal roll-off will exist and the resolution will decrease. This can be countered by increasing the length of the natural time constant τ_0 , which is achieved by making G small.



Figure 2.8. The top plot shows the spectral density on the TES input for $\alpha = 1$. Included is the TES Johnson noise (\bigcirc), the phonon noise (\times) and the signal (\diamondsuit). Also shown is the total noise spectral density (-). The bottom plot is for the case of $\alpha = 100$.

Changing G does not change the intrinsic resolution it only shifts all the curves to a different frequency roll-off. Keeping G small and making α big is the best way to optimise the *SNR* and hence the resolution, whilst keeping the signal response at frequencies lower than the diffusion and thermalisation processes. In a non-ETF-TES this would be problematic since the time constant is independent of α and decreasing G would decrease the device count rate even for a large α . For the ETF-TES a high count rate capability is possible by using a large α even though G may be small.

The FWHM energy resolution of the TES can be found from the detector *SNR* [Moseley, 1984] and is shown to be [Bruijn et al., 2000]:

$$\Delta E_{FWHM} = 2.36\xi \sqrt{k_b T^2 C} = 2 \left(\frac{1}{\alpha^2 L_0^2} + \frac{\gamma}{\alpha L_0} \right)^{1/4} \left(2.36 \sqrt{k_b T^2 C} \right) .$$
 2.42

Equation 2.42 only includes the TES Johnson noise and the phonon noise. The TES resolution depends on L_0 and α , maximising both optimises the resolution. From Equation 2.15 and 2.16, L_0 also depends on the bath temperature T_b and the transport mechanism between the TES and the cold bath n. For optimum energy resolution we require $(T_b/T)^n \ll 1$. The resolution is independent of the thermal conductance to the cold bath G. Changing G has the effect of also changing the required bias power P_0 by the same amount, such that L_0 remains unchanged. As was previously stated, for sufficiently large α devices the factor ξ can be as low as 0.1. For $\gamma = 0.5$, and large L_0 , this expression reduces to that derived by Irwin (1995b):

$$\Delta E_{FWHM} = 2.36\xi \sqrt{k_b T^2 C} = 2 \left(\frac{n}{2\alpha^2}\right)^{1/4} \left(2.36\sqrt{k_b T^2 C}\right).$$
 2.43

The additional noise sources from the shunt Johnson noise, read-out electronics or sources of TES 'excess' noise such as 1/f noise, internal thermal fluctuation noise [Hovers et al., 2000] or phase slip shot noise [Fraser, 2004], which can significantly degrade detector performance, could also be included to give a more complex expression. See for example the analysis by Feliciano-Figueroa (2001) for more details on noise and energy resolution. Further details of TES energy resolution will be described in Chapter 4.

As well as the limitation set by the thermalisation time there is a second practical constraint on how much improvement in resolution can be made by increasing α . The parameter α , along with the heat capacity *C* and the photon energy *E*, define the change in resistance ΔR , for a given temperature increase:

$$\Delta R = \alpha \frac{R_0}{T_0} \frac{E}{C} \,. \tag{2.44}$$

It is possible that ΔR maybe sufficiently large as to saturate the TES (or in other words drive the TES completely into the normal state). If saturation occurs some of the absorbed energy may not be measured by the TES and the resolution will be degraded. Thus the ratio C/α must be big enough for a given energy *E* to ensure that saturation does not occur. If we assume simply that the bias point is at $R_0 = R_n/2$, then for saturation to be avoided we require $\Delta R < R_n/2$. Thus, substituting for ΔR and R_0 in Equation 2.44 and re-arranging; we find a limit on how small C/α can be for a practical TES detector:

$$\frac{C}{\alpha} \ge \frac{E}{T_0}.$$

Substituting for C/α into Equation 2.43 we find the energy resolution limited by the saturation conditions is:

$$\Delta E_{sat} = 2.36 \sqrt{k_b T E \sqrt{8n}} \quad . \tag{2.46}$$

So although the intrinsic energy resolution of a microcalorimeter is independent of energy (for a given set of design parameters), to avoid saturation, the detector design parameters have to be scaled such that the resolution will scale as \sqrt{E} (like a CCD or STJ). It is also worth noting that α is a function of temperature throughout the transition edge, thus if ΔR is large, α will not be constant over the duration of the pulse. This non-linearity in detector response may act to reduce the detector resolution and again illustrates that a small C/α does not always lead to optimum resolution.

2.3. Summary

In this Chapter we have discussed the theory of microcalorimeter detectors, including semiconductor thermistors, Transition Edge Sensors and Metallic Magnetic Calorimeters. We have reviewed the detailed theory of the ETF-TES, which will be the basis of the rest of this thesis.
Chapter 3: The DROID Detector – Modelling the Position Response and Noise Characteristics

In this Chapter the concept of position-sensitive TES detectors is described. The respective advantages and disadvantages of TES pixel array detectors are outlined, followed by an introduction to the Distributed Read-Out Imaging Device or DROID, pursued at the University of Leicester, under development as an alternative to the pixellated design. In order to fully understand and characterise the DROID concept, we establish a finite-element model (FEM) that is capable of predicting not only the position response of the device, but also the noise characteristics of the dual TES system.

3.1. Position-Sensitive TES Detectors

3.1.1. Pixel Arrays

The challenge for TES detector development is to tile single pixel TESs into larger format arrays suitable for the next generation of X-ray telescope as described in Chapter 1. Kilbourne (2004) reviews the present state of microcalorimeter array development. The cryogenic imaging spectrometer baselined for the XEUS mission, outlined in Chapter 1, requires an $8 \times 8 \text{ mm}^2$ detector area, or in other words a 32×32 array of 250 µm single pixel TESs. The University of Leicester is part of a SRON (Space Research Organisation Netherlands) led consortium developing pixel arrays for XEUS. The Leicester role is in the signal processing and testing. Figure 3.1 shows a schematic of a 3-pixel test array developed by the SRON group, which is based on a Ti/Au TES with a Cu/Bi absorber. Prototype 5×5 pixel arrays have been tested with results showing full width at half maximum (FWHM) energy resolutions of 6-7 eV at 5.9 keV [Bruijn et al., 2004], somewhat worse than the best single pixel performance discussed in Section 2.1.3. Similar performance is reported for devices being developed by NASA for the Constellation-X mission [Lindeman et al., 2004a].



Figure 3.1. Vertical cross-sectional schematic diagram of a SRON 3-pixel test array. This device consists of Ti/Au TESs with Cu/Bi absorbers [Bruijn et al., 2004].

In such a pixellated detector the spatial resolution is limited only by the dimensions of the absorber pixels, the count rate capability is high (> kHz) since events are distributed across 1024 pixels in the array and the energy resolution should not be degraded from that of a single pixel TES. There are however, great challenges still to be overcome in the development of these large format arrays. For example, each of the 1024 TESs needs its own bias current resulting in 1024 times more power dissipated into the array substrate. Complex thermal engineering is therefore required to achieve close packing (a high fill-factor) without thermal cross-talk, while maintaining a bath temperature which is very much less than the transition temperature of the TES, as required for operation under extreme ETF conditions. Furthermore, in such arrays there is a need for a high density of electrical wiring, without electrical cross-talk. Complex SQUID multiplexing [Irwin, 2002] is required for the simultaneous read-out of all the 1024 pixels. Thus a pixel array cannot be considered a collection of individual single pixel detectors but is a complete unit with its own thermal and electrical requirements and must be designed as such. There is no doubt that the large format pixel arrays offer an impressive solution to the X-ray astronomy requirements discussed in Chapter 1, if these technical difficulties can be overcome.

Microcalorimeter arrays are also being developed for different photon wavelengths and other applications. The Stanford microcalorimeter group is developing 6×6 TES array for optical photometry of celestial objects. Current results show 0.15 eV at 1 eV [Cabrera et al., 2002]. Bolometric arrays are also being developed for ground based sub-mm radiation detection [Audley et al., 2004; Benford et al., 2004]. Figure 3.2 shows a test wafer from the SCUBA-2 instrument, which is a TES array for the James Clerk Maxwell Telescope (Hawaii), operating at sub-mm wavelengths [Audley et al., 2004]. In general the advantages and disadvantages of pixel arrays hold true for these bolometric detectors as well, except that the requirement for the precise measurement of energy is not as demanding as that for single photon detectors. Thus, it is possible to achieve large arrays without

compromising performance. In the SCUBA-2 example the aim is for an array of over 10000 TES detectors.



Figure 3.2. SCUBA-2 sub-array (40 × 40 pixel) test wafer [Holland, 2003].

3.1.2. DROID Concept

The alternative approach to the conventional pixel arrays is to use a position-sensitive detector (reviewed by Figueroa-Feliciano (2004)), incorporating two or more TESs for a single distributed absorber and using "heat diffusion" as the position-sensing principle. The total energy of an absorbed photon is estimated from the sum of the TES signals and the event position can be inferred from the difference in signals – thus the absorber exhibits position dependence. Position-sensitive imaging detectors offer a simpler engineering solution to the single pixel counterparts but with the same spatial coverage and without the complex read-out requirements. Several groups are now pursuing this alternative distributed approach [Cabrera et. al., 2004; Smith et al., 2004; Ohno et al., 2004; Iyomoto et al., 2006]. We are developing a linear detector called a Distributed Read-Out Imaging Device or DROID (see Figure 3.3) – an acronym originally coined by ESTEC researchers for position-sensitive STJ based detectors [den Hartog et al., 2000]). Our DROIDs consist of two TESs at either end of a continuous absorber that is 250 μ m wide. An array of 32 such, 8 mm long devices stacked together, would be equivalent to a 32 × 32 pixel array proposed for XEUS but requiring only 64 read-out channels with 16 times less power dissipation in the detector substrate.



Figure 3.3. Schematic diagram of the DROID concept.

The NASA Goddard Space Flight Centre (GSFC) group are also developing a distributed device called a Position-Sensitive TES or PoST [Figueroa-Feliciano et al., 2000]. The segmented PoST utilises a series of 250 μ m Au absorbers connected in a chain by small thermal links, with TES read-out at both ends of the chain. In this device, instead of a continuous distribution of pulse shapes, the pulses are discretely quantised depending upon the number of segmented pixels. This may be advantageous from the point of view of data analysis, since there are a discrete number of pulse shapes to differentiate between, rather than a continuous distribution as in the DROID concept. It also means that along the axis of the device the dimensions of each pixel limit the position resolution. The latest performance from a 7-pixel (2 mm) PoST show an impressive FWHM energy resolution of ~ 8-9 eV across the device at 5.9 keV [Iyomoto et al., 2006]. NASA GSFC are also developing Continuous PoST devices, similar to the Leicester DROIDs, which use a continuous linear X-ray absorber instead of the segmented absorber. These GSFC devices, originally designed to study the thermal properties of different absorber materials, have also demonstrated promising results with $\Delta E \sim 16$ eV FWHM at 4.5 keV at the centre of the device and 33 eV at the ends [Iyomoto et al., 2006]. Improved spectral performances should be possible for fully optimised devices.

The disadvantages of the large absorber devices are, firstly, a lower global count-rate. This will arise because of the ability to resolve only a single photon in the large absorber at any one time (pile-up) thus increasing the absorber size will decrease the count rate capability. Increasing the length of our absorber decreases the thermal conductance G_{abs} and increases the heat capacity C_{abs} , thus the absorber time constant ($\tau_{abs} = C_{abs}/G_{abs}$) will also be increased. If the absorber conductance is sufficiently large compared to the effective TES-bath thermal conductance ($G_{abs} > G_{eff}$, recalling that G_{eff} is defined by Equation 2.18), then the decay time constant can be approximated to ~ $C_{tot}/2G_{eff}$, where C_{tot} is the total heat capacity of the DROID including both TESs and absorber.

This is not the complete story since the overall time constant for the pulses will also be influenced by the TES-bath time constant, but in general increasing the absorber length will decrease the count-rate. The second disadvantage is again related to the fact that a larger absorber will have a higher heat capacity. Since the energy resolution scales as the square-root of the total heat capacity, we would expect a degraded energy resolution. The ideal DROID design will therefore have a heat capacity dominated by the two TESs, which must be fixed to avoid saturation (see Section 2.2.3), so the DROID energy resolution will ultimately be ~ $\sqrt{2}$ worse than the single pixel case. The energy and position resolving powers are, to first order, approximately equal: $E/\Delta E \sim L/\Delta x$ [Fraser, 1987]. Furthermore, ΔE is independent of E but Δx varies inversely with E. Therefore, for typical devices in the keV energy range, which have L of the order mm, the position resolution should be of the order 10^1 - 10^2 µm, comparable with the single pixel design. This is not the complete story since, as we will show in Chapter 4, the thermal conductance of the absorber and the TES-to-bath conductance are also of key importance for both energy and position resolution and can change both ΔE and Δx significantly from that predicted in the above approximation. However, with careful design, the performance of the DROID can be made comparable, if not superior, to pixel arrays, depending upon application.

3.2. Modelling Techniques for Position-Sensitive Cryogenic Detectors

In order to fully understand and characterise the DROID concept we require a model that is capable of predicting not only the position response of the device to an absorbed photon (under electro-thermal feedback (ETF) conditions), but also the energy and spatial resolutions over the complete length of the device ($\Delta E(x)$, $\Delta x(x)$). We will start by considering previous models used for similar position-sensitive devices.

Fraser (1987) investigated the importance of thermal non-uniformity in limiting the energy resolution of a position-sensitive calorimeter, consisting of two thermistors either side of a single absorber. The approach by that author is to solve the one-dimensional heat-diffusion equation [Carslaw and Jaeger, 1959] for a given deposition of energy into the system by application of the required boundary conditions. We have updated this model [Smith et al., 2004] to incorporate the effects of ETF in our TES based DROID devices. The resulting thermal pulse shapes are used to demonstrate the spatial variation in the pulse height and rise time across the length of the DROID absorber, confirming initial X-ray results (though the effects of the SQUID read-out and bias circuitry on the response are not incorporated). Hanna et al. (1996) also solve the heat diffusion equation for their microcalorimeter

consisting of a $100 \times 100 \ \mu\text{m}^2$ gold absorber. They demonstrate that the absorber exhibits position dependence and consequently measurement of only pulse height causes degradation in energy resolution. They conclude that a correction based on rise time can be made to deduce the original energy and location of the absorbed photon. Ashby et al. (2002) used a finite-element model to describe the thermal response of a cryogenic detector consisting of four TES sensors at the corners of a rectangular absorber. They also demonstrate analytical and neural network based techniques for determining the position of an absorbed photon - though again the effect of bias circuitry is not included. The above mentioned models are based on the assumption that the dominant thermal transport is one of diffusion only. Whether this is the case will depend upon the material, dimensions and purity of the absorber used – though this should be an adequate assumption for thin metallic films such as gold or copper where electron transport will dominate (see Chapter 6). For ballistic transport, a Monte Carlo technique is more suited, as used by Irwin (1995b), to model ballistic phonon propagation in a silicon crystal TES absorber. Trowell et al. (2002) uses a similar Monte Carlo technique to model the thermal response of DROID devices consisting of a bismuth absorber. In this case the author makes the assumption that the dominant transport mechanism is again in the form of ballistic phonons. The final model we consider is that by Figueroa-Feliciano (2001), who uses a finite-element technique to model the response of the NASA GSFC segmented PoST devices. This model is derived by representing each absorber element plus thermal link as a differential equation. The complete device is then represented by a set of coupled differential equations, which can be numerically solved to find the system response. By considering the associated sources of noise in the system, Figueroa-Feliciano (2001) also shows how the energy resolution for the device can be found for absorption in each individual pixel.

3.3. Modelling the DROID Position Response

To model the complete position response of the DROID we will use a one-dimensional finite-element approach as used by Figueroa-Feliciano (2001) for the GSFC PoST devices. Using this approach the continuous absorber can be represented as a series of heat capacities linked by thermal conductances. Ideally, the continuous nature of the DROID absorber will be well represented by using as larger number of elements as is possible. However, practical constraints on computer processing time mean that in reality the model density must be limited. Choosing the number of elements to use is thus driven by the trade-off between numerical accuracy and computation time. In appreciation of this we choose 21 nodal points to represent the DROID absorber. Figure 3.4 shows a schematic diagram of the Thevenin equivalent electrical bias and thermal circuits used to represent the DROID detector; only one end of the DROID is represented. This is identical to the single pixel TES thermal and electrical circuit (see Figure 2.6) except for the addition of the absorber elements.



Figure 3.4. Thevenin equivalent electrical bias circuit coupled to the thermal circuit of the DROID. V_{nl} is the Johnson noise from the load resistor and V_{nt} is the Johnson noise from the TES. N_{abs} is the number of absorber elements. Only the first three absorber nodes are shown. P_{nabs} is the noise from each thermal link.

Each $250 \times 250 \ \mu\text{m}^2$ TES, with heat capacity *C* and temperature T_C , is represented by a single element in the model, with the assumption that the resistance is a function of both temperature and current. In addition to the thermal circuit, we incorporate the effect of the electrical bias circuitry with SQUID input coil inductance L_{in} and load resistor R_l , resulting in two differential equations for each TES. We represent each absorber element as a heat capacity C_{abs}/N_{abs} with thermal conductance $2N_{abs}G_{abs}$ either side. Thus inter-absorber element conductances are $N_{abs}G_{abs}$, except the first and last conductance (connecting the absorber to each TES) which is $2N_{abs}G_{abs}$. The complete one-dimensional DROID system is then represented by a set of 25 differential equations. Initially excluding noise sources, applying Kirchhoff's rules to the Thevenin equivalent bias circuit leads to electrical circuit response of the TES, Equation 3.1:

$$L_{in}\frac{dI}{dt} = V_b - I(t)R(T,I) - I(t)R_l.$$
 3.1

The time-dependent thermal behaviour of the TES is then defined by:

$$C \frac{dT_{tes}}{dt} = I(t)^2 R(T, I) - K(T_{tes}(t)^n - T_b(t)^n) - 2N_{abs}K_{abs}(T_{tes}(t)^{n_{abs}} - T_{abs1}(t)^{n_{abs}}) + P_{\gamma}, \qquad 3.2$$

and similarly for the first absorber node we have:

$$\frac{C_{abs}}{N_{abs}}\frac{dT_{abs1}}{dt} = -2N_{abs}K_{abs}\left(T_{abs1}(t)^{n_{abs}} - T_{tes}(t)^{n_{abs}}\right) - N_{abs}K_{abs}\left(T_{abs1}(t)^{n_{abs}} - T_{abs2}(t)^{n_{abs}}\right) + P_{\gamma}, \qquad 3.3$$

with similar expressions for the other absorber nodes. Equation 3.2 is the thermal power balance between Joule heating in the film (term *A*) and power lost through the substrate (term *B*) and to the first absorber element (term *C*). Similarly, Equation 3.3 represents the power balance for the first absorber element. Term *D* is the power flow from the absorber to the TES and *E* is the power flow to the adjacent absorber element. All thermal power flow terms are expressed as power laws where *n* is the exponent and *K* is a constant with units W K⁻ⁿ (Equation 2.9). P_{γ} is the power input due to the photon absorption.

By applying a Taylor expansion and retaining only first order terms, this system of equations can be linearised for small changes about the operating point (ΔI , ΔT_{tes} , ΔT_{abs1} , ΔT_{abs2} , ... etc). Equations 3.4-3.6 are the final linearised equations:

$$L_{in} \frac{d\Delta I}{dt} = -\left[R_I + R_0 \left(1 + \alpha_I\right)\right] \Delta I - \frac{\alpha V_0}{T_C} \Delta T_{tes}, \qquad 3.4$$

$$C\frac{d\Delta T_{tes}}{dt} = V_0 \left(2 + \alpha_I\right) \Delta I + \left[-2N_{abs}G_{abs} - G + \frac{P_0\alpha}{T_C}\right] \Delta T_{tes} + 2N_{abs}G_{abs}\Delta T_{abs1} + P_{\gamma}, \qquad 3.5$$

$$\frac{C_{abs}}{N_{abs}}\frac{d\Delta T_{abs1}}{dt} = 2N_{abs}G_{abs}\Delta T_{tes} - 3N_{abs}G_{abs}\Delta T_{abs1} + N_{abs}G_{abs}\Delta T_{abs2} + P_{\gamma}.$$
3.6

For this small signal model we assume that C, α ($= T/R(\partial R/\partial T)$) and α_I ($= I/R(\partial R/\partial I)$) are all constants and locally independent of temperature. V_0 and P_0 are the bias voltage and power at the operating point. Equation 3.4 and 3.5 are identical to those for the single pixel TES (Equation 2.29 and 2.30) except for the additional heat loss terms between the TES and the absorber in Equation 3.5.

The resulting differential equations are then solved for ΔI (or ΔT) by numerical integration using the fourth-order Runge-Kutta method [Stroud, 1996], implemented in the programming language IDL. The initial conditions are in the form of an increase in temperature assuming a delta function input of energy ($\Delta T_0 = E/C$) into the TES/absorber thermal system. This allows the DROID response to be determined for photon absorption in any one of the 23 nodes.

Parameter	Symbol	DROID 1	DROID 2
Absorber length	l (mm)	4.32	4.75
Absorber width	w (µm)	250	250
Absorber thickness	<i>d</i> (nm)	210 ± 5	210 ± 5
Total detector length	L (mm)	4.82	5.25
TES heat capacity	$C (pJ K^{-1})$	1	1
Absorber heat capacity	C_{abs} (pJ K ⁻¹)	2.7	3.0
Absorber thermal conductance	G_{abs} (nW K ⁻¹)	2.34	2.13
Thermal conductance - TES to bath	$G(nW K^{-1})$	2.74 ± 0.46	4.33 ± 0.47
Thermal exponent	п	2.63 ± 0.09	3.27 ± 0.05
Conductance constant - TES to bath	$K (nW K^{-n})$	17.9 ± 2.5	68.7 ± 6.1
Absorber - bath conductance ratio	$g = G_{abs}/G$	0.85	0.49
Electrical resistance of the TES	$R_N(\mathrm{m}\Omega)$	758 ± 5	754 ± 4
TES sensitivity	α	30	30
TES transition temperature	T_C (mK)	176 ± 5	177 ± 5
Loop gain	L_0	11.4	9.2
Effective TES-bath time constant	$\tau_{eff}(\mu s)$	40.5	31.0
Absorber time constant	$ au_{abs}$ (µs)	1154	1408

Table 3.1. Key DROID parameters for the two different configurations.

Table 3.1 shows the key parameters for two different DROID configurations. The determination of some of these parameters is described in a Chapter 6. Figures 3.5 and 3.6 show, respectively, the pulse shapes for just TES 1 and both TES signals combined, for absorption of a 5.9 keV photon in every second thermal node. The parameters are for DROID 1 with $L_{in} = 100$ nH, $R_0 = 0.2 \Omega$, $\alpha_I = 0$ and $R_l = 35 \text{ m}\Omega$. In both cases, the largest (and fastest) pulse corresponds to absorption in the TES itself. Note that for clarity, these pulse shapes are inverted. Similarly, Figures 3.7 and 3.8 show, respectively, the pulse height and rise time variations across the detector for each absorber location. We see that the pulses become slower and smaller the further away from the TES the absorption occurs, thus demonstrating the spatial variation along the DROID that allows us to determine the event position. This is consistent with the original theory behind the DROID concept and consistent with our previous thermal model [Smith et al., 2004], based on the heat diffusion equation. The log plots show that after the initial spatially varying thermalisation, the pulses decay away at the same exponential rate. For DROID 1 the time constant fitted to the tail of the pulse is $\tau = 181 \ \mu s$ and similarly for DROID 2 we have $\tau = 191 \ \mu s$. Ironically, the second configuration, which aimed for higher count rate, has a slower decay time constant due to the longer absorber length (smaller G_{abs} and larger C_{abs}), despite the conduction to the heat bath G being higher for the DROID 2 design. Note that the experimentally measured X-ray response of a DROID detector (read out using only a single TES) is reported in Section 7.1.2.



Figure 3.5. Modelled pulse shapes using read-out at TES 1 for absorption in every second thermal node. The inset is a semi log plot of the same data confirming the exponential decay. The largest, fastest pulse corresponds to absorption in TES 1.



Figure 3.6. Modelled pulse shapes when the signals from TES 1 and TES 2 are added together. Inset is a semi log plot of the same data confirming the exponential decay.

To find the energy of the incident photon we must add the two TES signals. It is clear from Figure 3.7 and 3.8 that, in this particular case, the pulse height and rise time of the combined signals are highly position dependent and as such, cannot solely be used to estimate the photon energy. Whether this is the case or not will depend on the DROID design and in particular, the thermal conduction of the absorber G_{abs} . We can estimate the photon energy by integrating the combined pulse shapes, or using optimal filtering as described in Chapter 4. Figure 3.9 shows the estimated energy as a function of position for each independent TES signal and the combined signal from integrating the pulse shapes. Note that, unlike with pulse height, the total estimated energy is independent of position and thus allows us to conclusively determine the photon energy. Recall from Chapter 2 that only in the extreme ETF limit, with perfect voltage bias ($R_I = 0 \Omega$) and no current dependence ($\alpha_I = 0$), will the estimated energy equal the true energy. Figure 3.10 then gives the normalised energy ratio ($A_E = E_1/(E_1+E_2)$) which contains the position information. We will investigate the position determination and resolution in Chapter 4.



Figure 3.7. Modelled pulse height as a function of absorption position. The black line (\diamondsuit) is the pulse height of the combined signals and the red lines (\ast) are for each separate TES signal. The two TESs are situated at nodes 0 and 22.



Figure 3.8. Modelled pulse rise time as a function of absorption position. The black line (\diamondsuit) is the rise time of the combined signals and the red lines (*) are when using only 1 TES signal. The two TESs are situated at nodes 0 and 22. The pulse rise times are independent of photon energy.



Figure 3.9. Estimated pulse energy as a function of absorption position, demonstrating that the estimated energy is independent of position. The black line (\diamond) is the energy of the combined signals and the red lines (*) are for each TES separately. The two TESs are situated at node 0 and 22.



Figure 3.10. Normalised energy ratio as a function of absorption position. It is this function which allows us to determine the absorption position of the photons. The two TESs are situated at node 0 and 22.

3.4. Investigating the DROID Noise Response

As with a single pixel TES (see Section 2.2.3), the ultimate DROID performance will be determined by the intrinsic sources of noise in the system. These sources of noise are summarised in Table 3.2. A DROID will be subject to Johnson noise in the two TESs and load resistors, and phonon noise due to the thermal conduction from the TESs to the bath but also an additional noise source from thermal conduction in the absorber. Note that as the absorber is assumed to be isothermal, we set $\gamma = 1$ in Equation 2.38. These noise sources are intrinsically white and as such, any frequency dependence of the noise spectrum will come from the DROID itself. Other sources of TES noise such as 1/f noise, internal thermal fluctuation noise [Hoevers et al., 2000] or phase slip shot noise [Fraser, 2004] could also be included at this stage but are ignored for this initial analysis.

In determining the energy of an event, the data from each TES (including the signal and the noise) is added together. The noise sources discussed above will be sensed on both TESs, coupled by thermal conductance of the absorber. Thus, when the data is added, a correlation will exist. Understanding this correlation is fundamental to determining the performance of the DROID.

Noise	Form	Units
Johnson Noise (TES)	$V_{nt} = \sqrt{4k_b T_{tes} R_0}$	V Hz ^{-1/2}
Johnson Noise (load)	$V_{nl} = \sqrt{4k_b T_l R_l}$	V Hz ^{-1/2}
Phonon noise (TES-Bath)	$P_n = \sqrt{4 \varkappa_b T_{tes}^2 G}$	W Hz ^{-1/2}
Phonon noise (absorber end links)	$P_{nabs} = \sqrt{4k_b T_{tes}^2 2N_{abs} G_{abs}}$	W Hz ^{-1/2}
Phonon noise (all other absorber links)	$P_{nabs} = \sqrt{4k_b T_{tes}^2 N_{abs} G_{abs}}$	W Hz ^{-1/2}

Table 3.2. Sources of noise in the DROID detector.

The noise sources in Table 3.2 can be included into Equations 3.4-3.6 to produce:

$$\frac{d\Delta I}{dt} = \frac{1}{L_{in}} \begin{bmatrix} R_l + R_0 (1+\beta) \end{bmatrix} \Delta I - \frac{\alpha V_0}{L_{in} T_C} \Delta T_{tes} + (V_{nt} + V_{nl}) \frac{1}{L_{in}}, \qquad 3.7$$

$$\frac{d\Delta T_{tes}}{dt} = \frac{V_0}{C} (2+\beta)\Delta I + \left[-2N_{abs}\frac{G_{abs}}{C} - \frac{G}{C} + \frac{P_0\alpha}{CT_C}\right]\Delta T_{tes} + 2N_{abs}\frac{G_{abs}}{C}\Delta T_{abs1} + \dots \\ \dots (-IV_{nt} + P_n + P_{nabs1})\frac{1}{C}, \qquad 3.8$$

$$\frac{d\Delta T_{abs1}}{dt} = 2N_{abs}^2 \frac{G_{abs}}{C_{abs}} \Delta T_{tes} - 3N_{abs}^2 \frac{G_{abs}}{C_{abs}} \Delta T_{abs1} + N_{abs}^2 \frac{G_{abs}}{C_{abs}} \Delta T_{abs2} - (P_{nabs1} - P_{nabs2}) \frac{N_{abs}}{C_{abs}}.$$
 3.9

Note that the TES Johnson noise is coupled into both the TES electrical and thermal systems so appears in both Equations 3.7 and 3.8. Absorber phonon noise is essentially occurring in the link between two nodes and as such will appear in equations for adjacent nodes with opposite sign. This system of differential equations can then be generalised as follows:

$$\frac{d\Delta}{dt} = A\Delta + Bu + Gn \,. \tag{3.10}$$

The system variables in the column matrix Δ are the incremental changes to the TES currents and the temperatures, (ΔI , ΔT_{tes} , ΔT_{abs1} , ΔT_{abs2} , ... etc); *u* is the photon signal in the form of a delta function input of power at each node. Matrix *A* controls the dynamics of the system response whereas matrices *B* and *G* describe how the signal and noise, respectively, are coupled into the system. Matrix *A* is a 25 × 25 square matrix where the constants *a*-*f* are the bracketed terms as labelled in Equations 3.7-3.9. Not all elements in the matrices are shown since rows 4-22 repeat though the matrix, shifting by one column to the right with each increasing row number:

similarly matrix *B* is 25×25 square matrix:

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & & & \\ 0 & C^{-1} & 0 & 0 & \cdots & & & \\ 0 & 0 & N_{abs}C_{abs}^{-1} & 0 & \cdots & & & \\ 0 & 0 & 0 & N_{abs}C_{abs}^{-1} & \cdots & & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ & & & & N_{abs}C_{abs}^{-1} & 0 & 0 & 0 \\ & & & & 0 & N_{abs}C_{abs}^{-1} & 0 & 0 \\ & & & & 0 & 0 & C^{-1} & 0 \\ & & & & 0 & 0 & 0 & 0 \end{pmatrix},$$

G is a 28×25 matrix containing the noise coefficients:

$$G = \begin{pmatrix} L_{in}^{-1} & L_{in}^{-1} & 0 & 0 & \dots & & \\ -IC^{-1} & 0 & C^{-1} & C^{-1} & \dots & & & \\ 0 & 0 & 0 & -N_{abs}C_{abs}^{-1} & \dots & & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ & & & & N_{abs}C_{abs}^{-1} & 0 & 0 & 0 \\ & & & & & \dots & N_{abs}C_{abs}^{-1} & 0 & 0 & 0 \\ & & & & & \dots & 0 & 0 & L_{in}^{-1} & L_{in}^{-1} \end{pmatrix},$$

and finally *n* is a 1×28 matrix to incorporate the noise sources as listed in Table 3.2:

$$n = \begin{pmatrix} V_{nt} \\ V_{nl} \\ P_n \\ P_{nabs1} \\ P_{nabs2} \\ \vdots \end{pmatrix}.$$

To derive the theoretical noise spectral density on the outputs we need to take the Fourier transform of Equation 3.10 (replace d/dt by $i2\pi f$), to operate in the frequency domain. Taking the Fourier transform of Equation 3.10 and re-arranging to find the system variables Δ we have:

$$(i2\pi fI - A)\Delta = Bu + Gn$$

$$\Delta = (i2\pi fI - A)^{-1}Bu + (i2\pi fI - A)^{-1}Gn$$

$$\Delta = S(f)u + R(f)n,$$
3.11

where S(f) and R(f) are then defined as the signal and noise responsivities respectively, and I is a unit matrix. This differs slightly from the nomenclature used by Figueroa-Feliciano (2001) where the author uses a single responsivity matrix to describe the system. By decoupling the signal and noise responsivities we can more readily derive mathematical expressions for the noise, energy resolution and spatial resolution of our detector.

The measured output on variable Δ_k for a noise input source *n* is then:

$$\Delta_k(f) = R_{k,j}(f)n_j, \qquad 3.12$$

with a similar expression for the measured output due to photon signal input. Here, the subscript k corresponds to the node where we want to sample the noise. For ease of notation the equations are ordered so that k = 1 corresponds to the measured current (due to the signal or noise) on TES 1 and k = 2 is the measured current on TES 2. The subscript *j* corresponds to a specific noise source i.e. j = 3 is the phonon bath noise for TES 1. Independent (uncorrelated) noise powers add linearly, so that the amplitudes add in quadrature. The individual noise sources listed in Table 3.2 are not correlated and therefore we square each contribution and then sum to find the total noise on the output. The total noise $\langle |N_1(f)|^2 \rangle$ at the output of TES 1 (equivalent to the variance on Δ_1) is therefore:

$$\left\langle \left| N_{1}(f) \right|^{2} \right\rangle = \sum_{j} \left| R_{1,j}(f) \right|^{2} \left\langle n_{j}^{2} \right\rangle, \qquad 3.13a$$

and similarly for TES 2 we have,

$$\left\langle \left| N_2(f) \right|^2 \right\rangle = \sum_j \left| R_{2,j}(f) \right|^2 \left\langle n_j^2 \right\rangle, \qquad 3.13b$$

and the correlation between the two outputs will be:

$$\langle N_1^*(f)N_2(f)\rangle = \langle N_1(f)N_2^*(f)\rangle^* = \sum_j R_{1,j}^*(f)R_{2,j}(f)\langle n_j^2\rangle,$$
 3.13c

the star indicating the complex conjugate. The combined noise when we add the two TES signals together is:

$$\left< \left| N_{1+2}(f) \right|^2 \right> = \sum_j \left| R_{1,j}(f) + R_{2,j}(f) \right|^2 \left< n_j^2 \right>$$

$$= \sum_{j} \left| R_{1,j}(f) \right|^{2} \left\langle n_{j}^{2} \right\rangle + \sum_{j} \left| R_{2,j}(f) \right|^{2} \left\langle n_{j}^{2} \right\rangle + 2 \sum_{j} R_{1,j}(f) R_{2,j}^{*}(f) \left\langle n_{j}^{2} \right\rangle$$
$$= \left\langle \left| N_{1}(f) \right|^{2} \right\rangle + \left\langle \left| N_{2}(f) \right|^{2} \right\rangle + 2 \operatorname{Re} \left\langle N_{1} N_{2}^{*}(f) \right\rangle.$$
3.14

and includes the noise from each TES and the covariance.



Figure 3.11. Calculated noise spectral density on TES 1. Included is the phonon-bath (Δ), load (\diamond) and TES Johnson noise (\Box) from TES 1, with spectra similar to that expected from a single pixel TES (see Section 2.2.3). The phonon-bath (\times), load Johnson (+) and TES Johnson (*) noise from TES 2 are all coupled through the DROID absorber and appear as thermal noise when read-out on TES 1. For clarity, only every second absorber phonon noise (no symbol) contribution is included, the top line corresponds to the conductance closest to the TES and bottom line is the furthest contributing conductance. The thick dashed line gives the total noise contribution from the absorber. The thick black line gives the total noise spectrum.



Figure 3.12. Current noise spectral density for when TES 1 and 2 signals are added together. Due to the symmetry of the DROID some noise terms are overlaid. The key is the same as in Figure 3.11.

Using Equations 3.13a and 3.14 we show the root-mean-squared (rms) current noise $\Delta I_{rms} = \sqrt{\langle |N(f)|^2 \rangle}$ measured on TES 1 in Figure 3.11 and the noise when both TES signals are summed in Figure 3.12. The model uses the parameters for DROID 1 listed in Table 3.1, with the inductance set to zero to reveal the intrinsic detector response. The data shows the expected spectra for the Johnson noise, suppressed by the ETF at low frequency, and the phonon noise, from TES 1 (see Figure 2.8). The TES 2 noise is coupled through the DROID absorber thus appears as a thermal noise spectrum on TES 1. The phonon noise from each contributing absorber element converge at low frequency (in the D.C. limit) as heat can flow freely unimpeded around the system. As the frequency is increased above ~ 100 Hz, the 22 individual absorber noise contributions start to spread since the associated time constants filter the noise differently depending upon position. In the limit $G_{abs} \rightarrow 0$ the two TESs are completely de-coupled and the noise on each TES will reduce to the form of that expected for a single pixel TES (Section 2.2.3).

Modelling of the noise spectrum reveals the frequency dependent correlations between the individual noise sources when sampled on each TES. In a situation analogous to a resistance-capacitance transmission line, both TESs in the DROID will sense all noise contributions via the coupling of the absorber; however, one TES will sense noise which is attenuated and phase shifted relative to the other. When the two signals are summed, a correlation will exist, the exact nature of which will

depend upon the number and value of reactive components, as well as the frequency. This is most clearly indicated in the absorber phonon noise, where at low frequency, the noise contributions tend to cancel. These correlations are quantified by the covariance $\langle N_1 N_2^*(f) \rangle$ of the data (Equation 3.13c). Figure 3.13 shows the covariance for the absorber phonon noise, where a negative correlation exists at low frequencies. The phonon noise in the absorber is heat moving between adjacent heat capacities, hence a noise fluctuation causing an increase in temperature in one direction, will affect a drop in temperature in the other direction. At low frequency the absorber thermalises as a unit, such that one TES will detect a drop in temperature while the other detects an increase, thus when the signals are added the noise effectively cancels. As the frequency is increased, a consequent change in phase will be observed at the TESs, and depending upon which node the noise contribution is from, a positive correlation can arise. So it continues until the noise is attenuated to a negligible level. The exact form of the correlations will depend upon the thermal conductances and heat capacities which make up the system.



Figure 3.13. Phonon noise covariance for the absorber elements. The top line corresponds to the links closest to the TESs. The covariances are symmetric about the centre of the DROID; hence some elements overlay.

As the absorber time constant is decreased ($\tau_{abs} = C_{abs}/G_{abs}$), the coupling between the two TESs increases and more of the heat can flow unimpeded through the absorber, thus promoting the negative correlation up to a higher frequency. As $\tau_{abs} \rightarrow 0$ the phonon noise will completely cancel over all frequency space.

Figure 3.14 shows the covariance for the TES Johnson noise, phonon noise and load resistor Johnson noise. The TES-bath phonon and load Johnson noise is explained by a similar argument as that used to explain the absorber phonon noise. At low frequency the positive correlation is because the phonon bath noise measured at TES 1 will be sensed on TES 2 in phase, so when combined the two contributions add. The positive, in-band, correlation of the phonon bath noise means that P_{n1} and P_{n2} are added linearly and not in quadrature. The phonon bath noise for a single pixel detector with bath conductance *G*, will therefore be the same as a DROID, with each TES-bath conductance of *G*/2.



Figure 3.14. Covariance for TES-bath phonon noise (top line), load resistor Johnson noise (middle) and TES Johnson noise (bottom).

The Johnson noise from the TES results in a negative correlation at low frequency. This is due to the effect of the ETF of the two TESs. An increase in current due to the Johnson noise in TES 1 will be suppressed but not completely cancelled by the ETF. However, this noise is also thermally coupled through the DROID absorber so is detected by TES 2 in the form of an increase in temperature and resistance. Because of the ETF in TES 2, the Joule power and hence the current are reduced. Consequently, the positive current fluctuation on TES 1 will induce a negative current fluctuation on TES 2, when combined, the noise will be in anti-phase and will cancel. As discussed previously, phase shifts in the system at higher frequency introduce oscillations into the covariance.

The total covariance between the two signals is shown in Figure 3.15. Whether the net covariance is positive or negative will depend upon the exact system parameters.



Figure 3.15. Covariance between the total measured noise on TES 1 and TES 2.

3.5. Summary

In this Chapter we have derived a one-dimensional FEM based on heat diffusion, which has allowed us to investigate the position response of the DROID detector to X-ray photons. The model considers an ideal case without incorporating non-linear effects, heat loss through the membrane, or the effects of pulse pile-up and baseline instabilities, all of which can affect the device response and resolution. The FEM enables the determination of the spectral density of the intrinsic detector noise on each TES output and illustrates the frequency dependant correlations between the two TES signals. The number of elements in this model is limited only by the available computational power (processor speed and memory) and could be extended to incorporate larger systems or other conductive processes such as electron-phonon coupling in the TES or heat loss from the absorber to the membrane. Equally, we could examine a two-dimensional DROID detector i.e. a square absorber with TES at each corner, or a one-dimensional DROID with a composite gold/bismuth absorber. We would have, however, to dramatically reduce the element density in the model to achieve comparable processing time.

In this Chapter we have set the foundations for the second stage of the modelling, where we investigate the energy and position resolution across the complete length of the DROID. Ultimately we will be able to use our model to optimise the DROID design for specific detector applications.

Chapter 4: Calculating the DROID Energy and Position Resolution

Using the DROID noise characteristics found in Chapter 3, it is possible to calculate the best attainable energy and position resolution along the length of the DROID. In Chapter 4 we introduce the concept of the optimal/matched filter, which is the standard technique used for determining the photon energy in TES detectors. We describe pre-existing methods of obtaining the energy and position of a photon in a position-sensitive detector and apply it to the DROID concept. We then show that these methods are non-optimum and derive algorithms for improved energy and position resolution performance. These algorithms include the effect of the noise correlation between the detector outputs to minimise the variance on the estimated photon energy and position. Using numerical simulations, we show that improved energy and position resolution estimates can be obtained. The simulations also reveal the trade-offs in changing the thermal conductances and heat capacities of the detector elements, which enables a detector design to be optimised for a particular application. The algorithms reported in this Chapter have been published in [Smith et al., 2006a,b].

4.1. Optimal Filtering and Energy Resolution

The output signal train from an X-ray microcalorimeter consists of pulses that are contaminated by noise. From these pulses we need to estimate the energy of the original X-rays as accurately as possible (in the case of the DROID we add the two pulses together). This can be achieved by passing the data through a filter, either a real-time analogue filter (RC filtering) [Nicholson, 1974], or for best results, by digital means using an optimal/matched filter [Szymkowiak et al., 1993]. The optimal filter is constructed to give the best achievable signal-to-noise ratio and hence the best achievable resolution of the detector.

4.1.1. Single TES Optimal Filtering and Energy Resolution

Consider first a single TES. The simplest method to derive the matched/optimal filter is by least squares minimisation of the difference between the estimated energy and the actual energy of the incident photon. In deriving the optimal filter we have to be aware of two assumptions. Firstly, the response of the detector must be linear; that is, the pulse shapes are independent of energy (implying the amplitude of the pulse is proportional to the energy). This may not always be true if we consider the large signal case where saturation effects and the variation in $\alpha(T)$ can introduce non-linearity into the detector response. The second assumption is that the noise must be stationary, i.e. it does not vary with time; again this is not strictly the case since the detector noise will vary depending upon location on the transition edge. However, in our small signal model we can assume that both these conditions are met. For optimal filtering under non-linear conditions and non-stationary noise, see for example the analysis by Whitford et al. (2004, 2005) and Fixsen et al. (2002, 2004).

If the noise is dependent on frequency i.e. is not white noise, we must construct the filter in the frequency domain and then transform it back into the time domain using the inverse Fourier transform. The raw data D from the TES consists of an X-ray pulse that is contaminated with noise. This, in the frequency domain, can be expressed as:

$$D(f) = HS(f) + N(f), \qquad 4.1$$

where S(f) is the normalised model pulse shape (or the signal responsivity), H is simply an amplitude scalar for the photon energy and N(f) is the noise. Minimising the variance on H is achieved by minimising (in the least squares sense) the difference between the noisy data, D(f), and the model pulse shape. Because the noise is, in general, not white i.e. the uncertainty on the measured signal at each frequency interval is not the same, weighted least squares minimisation is required (see Forbes et al. (2002) for example). The χ^2 condition to minimise is therefore [Szymkowiak et al., 1993]:

$$\chi^{2} = \int_{-\infty}^{\infty} \frac{\left(D(f) - HS(f)\right)^{2}}{\left\langle \left|N(f)\right|^{2} \right\rangle} df , \qquad 4.2$$

which is properly normalised because, substituting for D(f) (Equation 4.1) into Equation 4.2 we have: $\chi^2 = 1$.

Equation 4.2 is minimised by taking $d\chi^2/dH$, setting the result to zero and re-arranging to find H:

$$H = \frac{\int_{-\infty}^{\infty} \frac{D(f)S(f)^* + S(f)D(f)^*}{2\left\langle \left| N(f) \right|^2 \right\rangle} df}{\int_{-\infty}^{\infty} \frac{S(f)S(f)^*}{\left\langle \left| N(f) \right|^2 \right\rangle} df},$$

$$4.3$$

where superscript star terms are complex conjugates. The analysis used here and throughout this Chapter makes use of the symmetry properties of complex numbers, that is: $Z(-f) = Z^*(f)$, where Z is the Fourier transform of a real number. From this it follows that the integral of Z over all frequencies is real. Equation 4.3 can then be simplified to:

$$H = \frac{\int_{-\infty}^{\infty} \frac{D(f)S(f)^{*}}{\left\langle \left| N(f) \right|^{2} \right\rangle} df}{\int_{-\infty}^{\infty} \frac{S(f)S(f)^{*}}{\left\langle \left| N(f) \right|^{2} \right\rangle} df} = \frac{\int_{-\infty}^{\infty} D(f)F(f)df}{\int_{-\infty}^{\infty} \frac{\left| S(f) \right|^{2}}{\left\langle \left| N(f) \right|^{2} \right\rangle} df}.$$

$$4.4$$

The denominator in Equation 4.4 is a normalisation factor to give H in units of energy and,

$$F(f) = \frac{S^*(f)}{\left\langle \left| N(f) \right|^2 \right\rangle},$$
4.5

is then defined as the matched or optimal filter. In the frequency domain this is a weight function which, when multiplied by the signal, has the same effect as passing the signal through a filter in the time domain. Taking out the normalisation factor as a constant k, Equation 4.4 can be re-written as:

$$H = k \int_{-\infty}^{\infty} \frac{D(f)}{S(f)} SNR(f)^2 df .$$
4.6

where *SNR* is the signal-to-noise ratio. Conceptually, each frequency bin gives an independent measure of the photon energy. The estimates at each frequency are averaged with weights equal to the square of the signal-to-noise ratio. The optimal filter is derived from a minimisation problem thus the variations in filter shape about the optimum only cause small variations in the filtered data. The advantage of this is that the detector resolution is not strongly dependent upon determining the exact form of the filter. If the noise were white, all samples would have the same variance $(\langle |N(f)|^2 \rangle = constant)$ and the optimal filter would simply be the average of all the pulse shapes. For the more general case of non-white noise, S(f) is the average pulse shape and N(f) is found from averaging noise

spectra not containing a signal pulse. In the context of our model $\langle |N(f)|^2 \rangle$ and $|S(f)|^2$ are found from Equations 3.13 and 3.11 respectively.

If we substitute the raw data D(f) (Equation 4.1), back into Equation 4.4 and set our input energy to E, we have:

$$H = \frac{\int_{-\infty}^{\infty} (ES(f) + N(f))F(f)df}{\int_{-\infty}^{\infty} \frac{|S(f)|^2}{\langle |N(f)|^2 \rangle} df}$$
$$= E + \frac{\int_{-\infty}^{\infty} N(f)F(f)df}{\int_{-\infty}^{\infty} \frac{|S(f)|^2}{\langle |N(f)|^2 \rangle} df}.$$

If we take the time average of the above expression, the noise term averages to zero and as expected, the estimate of the energy is simply $\overline{H} = E$.

The energy resolution of the detector is then found from the variance (σ^2) on the energy estimate *H*. The definition of the variance is:

$$\sigma_{H}^{2} = \left\langle \int_{-\infty}^{\infty} (H - \overline{H})^{2} df \right\rangle.$$

$$4.7$$

This leads directly to:

$$\sigma_{H}^{2} = \frac{\int_{-\infty}^{\infty} \langle \left| N(f) \right|^{2} \rangle \left| F(f) \right|^{2} df}{\left| \int_{-\infty}^{\infty} \frac{\left| S(f) \right|^{2}}{\langle \left| N(f) \right|^{2} \rangle} df \right|^{2}} = \left(\int_{-\infty}^{\infty} \frac{\left| S(f) \right|^{2}}{\langle \left| N(f) \right|^{2} \rangle} df \right)^{-1}.$$

$$4.8$$

This is conventionally expressed with an integral range of between 0 and ∞ ; therefore the root-mean-squared (rms) energy resolution is:

$$\Delta E_{rms} = \sqrt{\sigma_H^2} = \left(\int_0^\infty \frac{4|S(f)|^2}{\left\langle \left| N(f) \right|^2 \right\rangle} df \right)^{-1/2}.$$
4.9

The extra factor four comes about from dividing the N^2 in Equation 4.9 by two, since the noise expressions in Table 3.2 are implicitly defined for frequency ranges from 0 to ∞ (not $\pm \infty$) and similarly the other factor of two comes from changing the integral range of Equation 4.8 to include positive frequencies only. Defining $\langle |N(f)|^2 \rangle / |S(f)|^2 = NEP^2$, where the NEP (W Hz⁻¹) is the noise equivalent power (the input referred power needed to produce a given output noise), Equation 4.8 is equivalent to that derived by Moseley et al. (1984):

$$\Delta E_{rms} = \left(\int_{0}^{\infty} \frac{4}{NEP^{2}(f)} df\right)^{-1/2}.$$
4.10

The lower the *NEP* the better the energy resolution of the detector. This is a general result for the case of a single pixel detector or when we consider the signal from only one TES in our DROID, and assumes that the optimal filter is known and available for every pulse. Analytical solution of Equation 4.10, considering only Johnson noise and phonon noise, leads to the derivation of the single pixel energy resolution as defined by Equation 2.42.

4.1.2. DROID Energy Resolution

To determine the energy of an absorbed photon in the DROID we must add the two TES signals together. If, when we combined the two signals, the resulting pulse shape was position independent, we would only need a single filter for every absorber position. We have seen, however, that for the current prototype DROID devices, even when both TES signals are added together, the resulting pulse shapes change with position (see Figure 3.6). If the pulse shapes are not the same, a unique filter will not be optimally matched to every pulse. Using this method we assume that the optimal filter is available for every interaction point i.e. the event position must be known, this can be found from the method outlined in Section 4.2. If we could increase the absorber conductance, the pulse shapes of the combined signals would tend towards a single shape and the optimum energy resolution could be achieved with a single filter. However, in doing so, we must consider that there would be less spatial variation in the pulse shape is what is needed to determine the event location, thus the correct G_{abs} must be chosen to optimise performance.

We will first consider the method used by Figueroa-Feliciano (2001) for the GSFC PoST; adding the noisy signals together and then applying a single optimal filter to the result. Using this approach the summed data from the TESs is:

$$D_1(f) + D_2(f) = H(S_1(f) + S_2(f)) + N_{1+2},$$

$$4.11$$

with the subscripts 1 and 2 corresponding to each TES. N_{1+2} is the total summed noise. The χ^2 parameter to minimise is therefore:

$$\chi^{2} = \int_{-\infty}^{\infty} \frac{\left(D_{1}(f) + D_{2}(f) - H\left(S_{1}(f) + S_{2}(f)\right)\right)^{2}}{\left\langle \left|N_{1+2}(f)\right|^{2}\right\rangle} df .$$

$$4.12$$

Using the same methodology as before, we have for the energy estimate:

$$H = \frac{\int_{-\infty}^{\infty} [D_{1}(f) + D_{2}(f)]F(f)df}{\int_{-\infty}^{\infty} \frac{|S_{1}(f) + S_{2}(f)|^{2}}{\langle |N_{1+2}(f)|^{2} \rangle}df},$$
4.13

where the optimal filter is now defined as:

$$F(f) = \frac{S_1^*(f) + S_2^*(f)}{\left\langle \left| N_{1+2}(f) \right|^2 \right\rangle}.$$
4.14

From the variance of Equation 4.13 (defined by Equation 4.7), we can again find the detector energy resolution:

$$\Delta E_{rms} = \left(\int_{0}^{\infty} \frac{4 |S_1(f) + S_2(f)|^2}{\left\langle |N_{1+2}(f)|^2 \right\rangle} df \right)^{-1}.$$
4.15

This is in the same form as Equation 4.9 but with the re-definition of $NEP^2 = \langle |N_{1+2}| \rangle / |S_1(f) + S_2(f)|^2$. We will show in Section 4.3 that this method does not provide the optimum solution under all conditions.

4.2. Position Resolution using the Normalised Energy Ratio

Let us consider now the position determination and resolution. Determining the event position in a DROID is analogous to the one-dimensional RC line position encoders using relative timing [Fraser et al., 1981a] or amplitude [Fraser et al., 1981b] information from two charge signals to encode position. Trowell et al. (2002) uses this normalised energy ratio technique to estimate the position resolution in one-dimensional TES DROIDs. Position sensitivity in one-dimensional STJ DROIDs is similarly achieved using the charge ratio from the two STJs $(Q_1-Q_2)/(Q_1+Q_2)$ [Kraus et al., 1989].

We will start in the first instance by using a similar method. We make the assumption that the position of an event is described by $x = f(A_E)$, where $A_E = E_1/(E_1+E_2)$; in this case E_1 and E_2 are the energy estimates from the two TES signals and are found from Equation 4.4 using optimal filtering (or by integrating the pulse shapes as described in Chapter 2). The amplitude ratio A_E could take a variety of forms ($A_E = E_1/E_2$ for example); however we will use A_E as defined above because it is a symmetric, well-behaved function that is easy to deal with within the model (whereas $A_E = E_1/E_2$ goes to ∞ at x =0).

The optimal filters used to determine E_1 and E_2 will depend on the position of the event, which initially, is unknown. An iterative process will therefore be required to accurately locate the position. In the first instance (as with the energy resolution estimate in Section 4.1) we will assume that the optimal filter is available for each TES signal, at every absorption position.

 Δx can be found by considering the propagation of errors [Sivia, 1997] from energy to position. Differentiating *x* with respect to A_E we have:

$$\delta x = \left(\frac{\delta x}{\delta A_E}\right) \delta A_E = \frac{\delta A_E}{S_E},$$
4.16

where $S_E = \delta A_E / \delta x$ is the position sensitivity and A_E is our amplitude ratio defined above. Figure 4.1a shows the calculated energy estimates and Figure 4.1b shows the calculated S_E and A_E for prototype DROID 1. This data results from the optimal filtering method of finding the photon energy. In a practical situation, both S_E and A_E will have to be determined in advance to actual operations, either by prior calibration or from modelled data.



Figure 4.1. a) Graph of E_1 , E_2 and $E = E_1 + E_2$ as a function of position, assuming 5.9 keV photons; b) amplitude ratio A_E corresponding to the left axis (black *) and sensitivity S_E corresponding to the right axis (red \diamond). The results are for DROID 1 derived using optimal filtering as described in Section 4.1.1.

Expanding δx we have:

$$\delta x = \frac{1}{S_E} \frac{(E_1 + E_2)\delta E_1 - E_1\delta(E_1 + E_2)}{(E_1 + E_2)^2},$$
$$S_E \frac{(E_1 + E_2)^2}{E_1 E_2} \delta x = \frac{\delta E_1}{E_1} - \frac{\delta E_2}{E_2}.$$

Squaring both sides and taking time-averaged (expectation) values gives:

$$\left(S_E \frac{\left(E_1 + E_2\right)^2}{E_1 E_2}\right)^2 \langle \delta x^2 \rangle = \frac{\langle \delta E_1^2 \rangle}{E_1^2} + \frac{\langle \delta E_2^2 \rangle}{E_2^2} - 2\frac{\langle \delta E_1 \delta E_2 \rangle}{E_1 E_2},$$

after rearranging, the position resolution becomes:

$$\Delta x = \frac{1}{S_E} \frac{E_1 E_2}{(E_1 + E_2)^2} \sqrt{\left(\left(\frac{\sigma_1}{E_1}\right)^2 + \left(\frac{\sigma_2}{E_2}\right)^2 - \frac{2\sigma_{12}}{E_1 E_2}\right)^2} .$$
4.17

 σ_1 and σ_2 are the errors associated with energy estimators E_1 and E_2 ; in other words, the energy resolutions for each TES signal taken using Equation 4.9. σ_{12} is the covariance and a measure of the correlation between the two signals as investigated in Chapter 3. The energy contribution due to this

noise covariance is found in the same way that we arrived at Equation 4.15 but this time we are applying different filters to each TES signal rather than combining the signal and then filtering. Therefore the energy resolution contribution from the covariance is:

$$\left\langle \delta E_{1} \delta E_{2} \right\rangle = \frac{\operatorname{Re}\left[\int_{0}^{\infty} \left\langle N_{1} N_{2}^{*}(f) \right\rangle F_{1}(f) F_{2}^{*}(f) df\right]}{4 \int_{0}^{\infty} \frac{\left|S_{1}(f)\right|^{2}}{\left\langle \left|N_{1}(f)\right|^{2} \right\rangle} df \int_{0}^{\infty} \frac{\left|S_{2}(f)\right|^{2}}{\left\langle \left|N_{2}(f)\right|^{2} \right\rangle} df},$$

$$4.18$$

where F_1 and F_2 are the optimal filters for each TES signal found using the methodology of Section 4.1.1. These optimal filters used to compute E_1 and E_2 are functions of x; therefore the calculation must be iterated to find the final value of x. The term σ_{12} can be positive or negative (depending on the DROID parameters); however, the contribution to the position resolution comes into Equation 4.17 as negative, therefore positive correlations decrease ΔE but increase Δx . By inspection of Equation 4.17 we see that importantly, $\Delta x \propto 1/E$ (where E is the photon energy), in contrast to a single pixel detector where Δx , defined by pixel dimensions, is independent of E.

Due the symmetry of the DROID at the centre of the device (x = L/2), the energy of the photon will be split equally between the two TESs, therefore; $E_1 = E_2 = E/2$. Similarly the energy resolution components will be the same; $\sigma_1 = \sigma_2$. We can also make the approximation that $\sigma_{12} \sim 0$. The amplitude sensitivity S_E will be variable depending upon the time constants of the system but can be reasonably approximated to 1/L to within a factor of a few. This leads to the approximation that at the centre of the DROID, the energy and position resolving power will be of the same order:

$$\frac{\Delta x}{L} \sim \frac{\Delta E}{E} \,. \tag{4.19}$$

Typically, it is this approximation which is used to design position-sensitive detectors and states that the Δx will scale proportionally with ΔE . We will show by numerical simulation in Section 4.4 that this can be a misleading assumption since Δx could be significantly worse at the ends of the device compared to the centre (depending upon the device design parameters).

4.3. Optimised Filtering for Position and Energy Determination

The method described in Section 4.1.2 for determining the energy resolution is not the optimum solution i.e. does not lead to the best achievable resolution over all parameter space. This is because it

attributes equal weighting to each TES signal, even though the contribution from each signal to the overall energy resolution may be very different. Neither is the position determination using the normalised energy ratios optimum, since it uses only energy information and not the information from the complete pulse shape. The optimum solution requires the application of different filters to the TES signals so that the total noise power is minimised. The true optimum is thus found by treating the two outputs as the components of a single (vector) signal.

Considering TES 1, the pulse shape S_1 can be expanded as a Taylor expansion about an assumed position *x*:

$$S_1(f, x_{\gamma}) = S_1(f, x) + \left(x_{\gamma} - x\right) \frac{dS_1(f, x)}{dx} + \dots,$$
4.20

where second and higher order terms are omitted. Here $dS_1(f,x)/dx = S_1/(f,x)$ is the (non-linear) function representing the variation in pulse shape with position, and $\delta x = (x_{\gamma} - x)$ is the difference between the actual interaction position x_{γ} and an initial estimated position x. Including the signal $HS_1(f)$ and noise $N_1(f)$, the data D_1 can then be expressed as:

$$D_1(f,x) = H\left(S_1(f,x) + \delta x S_1^{\prime}(f,x)\right) + N_1(f), \qquad 4.21$$

with a similar expression for the data at TES 2, $D_2(f,x)$. Thus in the limit $\delta x \to 0$, the exact interaction position is known and Equation 4.21 reduces to, $D_1(f) = HS_1(f) + N_1(f)$. If the noise measured at each TES was white and there existed no correlation between the noise on each TES, the χ^2 condition to minimise, expressed in matrix notation, would be:

$$\chi^{2} = \int_{-\infty}^{\infty} \left[D - H \left(S + \delta x S^{\prime} \right) \right]^{*T} \left[D - H \left(S + \delta x S^{\prime} \right) \right] df , \qquad 4.22$$

where superscript *T* is the matrix transpose and the vectors *D*, *S* and *S'* are column matrices: $D = \begin{pmatrix} D_1(f) \\ D_2(f) \end{pmatrix}$, $S = \begin{pmatrix} S_1(f) \\ S_2(f) \end{pmatrix}$ and $S' = \begin{pmatrix} S_1'(f) \\ S_2'(f) \end{pmatrix}$. Both *H* and δx are unknowns to be found to yield the

event energy and position. However, for the more general case where the noise is not white and there does exist a correlation between the noise sampled on each TES, Equation 4.22 does not lead to the best resolution. The noise can be expressed in terms of its variance-covariance matrix V:

$$V = \begin{pmatrix} \left\langle \left| N_1(f) \right|^2 \right\rangle & \left\langle N_1 N_2^*(f) \right\rangle \\ \left\langle N_1^* N_2(f) \right\rangle & \left\langle \left| N_2(f) \right|^2 \right\rangle \end{pmatrix},$$

$$4.23$$

where the diagonal elements are the noise powers measured at each TES (Equations 3.13a and b) and the off diagonal elements are the covariances (Equation 3.13c). Then, using generalised Gauss-Markov regression [Forbes et al., 2002], the χ^2 condition for the coupled, dual TES system is:

$$\chi^{2} = \int_{-\infty}^{\infty} \left[D - H \left(S + \delta x S^{\prime} \right) \right]^{*T} V^{-1} \left[D - H \left(S + \delta x S^{\prime} \right) \right] df , \qquad 4.24$$

where V^1 is defined as the weight matrix and is the inverse of the covariance matrix of Equation 4.23.

The position and energy determination methods in Sections 4.1 and 4.2 use the assumption that the correct optimal filters are known. This means that determining the energy requires knowing the position and vice versa. Consequently, energy and position are correlated, as are their uncertainties. To find the true theoretical optimum resolution, incorporating the effect of this correlation, Equation 4.24 must be solved simultaneously for *H* and δx . This is achieved by setting $\partial \chi^2 / \partial H = 0$ and $\partial \chi^2 / \partial \delta x = 0$. Firstly, differentiating χ^2 with respect to *H* and setting to zero we have:

$$\frac{\partial \chi^2}{\partial H} = 2 \int_{-\infty}^{\infty} \left[\left(S + \delta x S' \right) \right]^{*T} V^{-1} \left[D - H \left(S + \delta x S' \right) \right] df = 0.$$

$$4.25$$

As $\int S^{*T} V^{-1} S' = \int S'^{*T} V^{-1} S$, neglecting terms of the order ∂x^2 , expanding and re-arranging we have:

$$\int_{-\infty}^{\infty} HS^{*T}V^{-1}Sdf + \delta x \left(2\int_{-\infty}^{\infty} HS^{*/T}V^{-1}Sdf - \int_{-\infty}^{\infty} S^{/*T}V^{-1}Ddf \right) = \int_{-\infty}^{\infty} S^{*T}V^{-1}Ddf .$$

$$4.26$$

On the left hand side we can replace *D* with *HS* because the difference HS - D, is the noise, which is assumed to be small compared to *HS* itself. As this is multiplied by δx and we are looking for the variance at $\delta x = 0$, this approximation has only a second-order effect. We then have:

$$H\int_{-\infty}^{\infty} F^T S df + H \delta x \int_{-\infty}^{\infty} F^{T} S df = \int_{-\infty}^{\infty} F^T D df , \qquad 4.27$$

where the product $F^T = S^{*T}V^{-1}$ is defined as the two-element *optimal filter matrix*. When multiplied out the two filters become:

$$F_{1}(f) = \frac{S_{1}^{*}(f) \langle |N_{2}(f)|^{2} \rangle - S_{2}^{*}(f) \langle N_{1}N_{2}^{*}(f) \rangle}{\langle |N_{1}(f)|^{2} \rangle \langle |N_{2}(f)|^{2} \rangle - \langle N_{1}N_{2}^{*}(f) \rangle \langle N_{1}^{*}N_{2}(f) \rangle},$$

$$4.28a$$

and

$$F_{2}(f) = \frac{S_{2}^{*}(f) \langle |N_{1}(f)|^{2} \rangle - S_{1}^{*}(f) \langle N_{1}^{*}N_{2}(f) \rangle}{\langle |N_{1}(f)|^{2} \rangle \langle |N_{2}(f)|^{2} \rangle - \langle N_{1}N_{2}^{*}(f) \rangle \langle N_{1}^{*}N_{2}(f) \rangle}.$$
4.28b

Thus each filter applies different weighting to the signals depending upon the signal and the noise contributions at the two TESs, as well as the noise covariance between them. In the limit as $\langle N_1 N_2^*(f) \rangle \rightarrow 0$ (i.e. no covariance between the two TESs), both these filters reduce to the single TES form as defined in Section 4.1.1.

Similarly, for the position determination, we differentiate Equation 4.24 with respect to δx and set to zero:

$$\frac{\partial \chi^2}{\partial \delta x} = -2 \int_{-\infty}^{\infty} HS' V^{-1} \Big[D - H \Big(S + \delta x S' \Big) \Big] df = 0, \qquad 4.29$$

which after expanding and re-arranging leads to:

$$H\int_{-\infty}^{\infty} F^{T} S df + H \delta x \int_{-\infty}^{\infty} F^{T} S^{T} df = \int_{-\infty}^{\infty} F^{T} D df , \qquad 4.30$$

where F' = dF/dx. Thus we are applying the differential (with respect to x) of the filters F' to the data D(f), not the actual filters themselves. Equations 4.27 and 4.30 are two simultaneous equations in H and $H\delta x$ which have the solution:

$$\binom{H}{H\delta x} = \left[\int F^T Sdf \int F^{T} S' df - \left(\int F^{T} Sdf\right)^2\right]^{-1} \left(\int F^{T} S' df - \int F^{T} Sdf\right) \left(\int F^T Ddf \int F^{T} Ddf\right).$$

$$4.31$$

In the absence of noise, substituting for D = ES, where E is the photon energy we find:

$$H = E$$

$$H \delta x = 0$$

$$4.32$$

This is simply the statement of the facts that we have correctly estimated the photon energy *E* and have found the correct event position since $\delta x = 0$. In solving for *H* and δx we require in advance, the forms of F'^T , F^T , S' and *S* as a function of position; these can be determined either by simulated or prior calibration data.

To determine the energy resolution ΔE and the position resolution Δx , we must find the variance on H and δx . From Equation 4.31, this leads to the root-mean-squared (rms) energy resolution (see Appendix A for the full derivation of the variances):

$$\Delta E_{rms} = \sqrt{\sigma_{H}^{2}} = \frac{\left[\int_{0}^{\infty} F^{T} S' df\right]^{1/2}}{2\left[\int_{0}^{\infty} F^{T} S df \int_{0}^{\infty} F^{T} S' df - \left(\int_{0}^{\infty} F^{T} S df\right)^{2}\right]^{1/2}},$$
4.33

and similarly the position resolution:

$$\Delta x_{rms} = \sqrt{\sigma_{\delta x}^2} \frac{\left[\int_0^{\infty} F^T S df\right]^{1/2}}{2H\left[\int_0^{\infty} F^T S df\int_0^{\infty} F^{T} S^T df - \left(\int_0^{\infty} F^{T} S df\right)^2\right]^{1/2}}.$$
4.34

The last term in the denominators of Equations 4.33 and 4.34 is a measure of the correlation between ΔE and Δx . If we solved for *H* and δx independently of each other, this term would not be included. Due to symmetry, this term will tend to zero at the centre of the DROID (x = L/2) where $S_1(f) = S_2(f)$ and $F_1'(f) = -F_2'(f)$. From Equation 4.31 it is also possible to calculate the covariance between *H* and δx :

$$\sigma_{H\delta x} = \frac{-\int_{0}^{\infty} F^{T} S df}{4H \left[\int_{0}^{\infty} F^{T} S df \int_{0}^{\infty} F^{T} S^{T} df - \left(\int_{0}^{\infty} F^{T} S df\right)^{2}\right]}.$$

$$4.35$$

This again tends to zero at x = L/2. Figure 4.2 shows the correlation coefficient, defined as: $\rho = \sigma_{H\tilde{\alpha}x}/(\sigma_H\sigma_{\tilde{\alpha}x})$, for the DROID 1 configuration. The correlation between *H* and δx has the effect of slightly degrading both the position and energy resolution towards the ends of the device as per the last term in the denominator of Equations 4.33 and 4.34.



Figure 4.2. Correlation ρ between H and δx across the length of DROID 1.

We will show in Section 4.4 that the worst resolution (both ΔE and Δx) across the length of the detector will be at x = L/2. Thus, because the last term in the denominator of Equations 4.33 and 4.34 tends to zero at x = L/2, we can say the limiting energy resolution across the length of the DROID will be:

$$\Delta E_{x=L/2} = \left(4\int_{0}^{\infty} F^{T}Sdf\right)^{-1/2}.$$
4.36

This is equivalent to the single pixel definition from Equation 4.9, but with $NEP^2 = 1/(F^TS)$. Similarly, the limiting position resolution will be:

$$\Delta x_{x=L/2} = \left(4H^2 \int_{0}^{\infty} F^{/T} S^{/} df\right)^{-1/2},$$
4.37

with $NEP^2 = 1/(F^T S')$. The improvement over the methods outlined in Sections 4.1-4.2, is demonstrated in the next section by numerical simulation.
4.4. Numerical Simulations of Energy and Position Resolution

4.4.1. Energy Resolution

Figure 4.3 shows the calculated full width at half maximum (FWHM) energy resolution ($\Delta E = 2.355\Delta E_{rms}$) across DROID 1 for three cases: independent optimal filtering of each TES (Section 4.1.1), optimal filtering of the summed TES signals (Section 4.1.2) and the optimised filtering of Section 4.3. The optimised method requires the calculation of differentials with respect to *x*. These are calculated for each absorber node using the two adjacent nodes; the TESs are omitted from the calculation. Consequently, the resolution is not calculated for the TESs and the first and last absorber elements.



Figure 4.3. Calculated FWHM ΔE along the length of DROID 1. The blue line (*) is the resolution when both signals are added together and then an optimal filter applied to the result. The red lines (\diamond) show the resolution using optimal filtering of each TES signal separately. The black line (\Box) shows the resolution using the new optimised filtering method to find the photon energy. The two TESs are situated at nodes 0 and 22.

The rapid increase in ΔE with distance from each TES, when considering individual TES signals, reflects the decreasing *SNR* as the distance is increased. The optimised filtering shows slight improvements in ΔE near the ends of the DROID (nodes 0-6 and 16-22), where $S_1(f)$ and $S_2(f)$ are significantly different. Both methods approach the same value for ΔE at the centre of the DROID where $S_1(f)$ and $S_2(f)$ are identical. Without the effect of the correlation between *H* and δx , as is expected, the resolution using the optimised filtering tracks very closely the single TES energy resolution at the device ends (and in fact is slightly better than the single TES resolution because of the noise covariance $\langle N_1 N_2^*(f) \rangle$).



Figure 4.4. Spectral density, of the DROID 1 response to 5.9 keV photons for absorption in each node, due to symmetry some lines are overlaid. For comparison, the total noise spectral density is also shown. The high frequency SNR, and hence ΔE , is worse at the centre of the DROID.

Figure 4.4 shows the frequency response of the summed signal for absorption in each node of DROID 1 (equivalent to $(|(ES_1(f) + ES_2(f))|^2)^{1/2})$. Also shown is the total noise spectral density. For events absorbed at the centre of the device, the high frequency components of the measured signal are more heavily attenuated by the absorber, compared to events absorbed closer to the ends of the device. Consequently, the *SNR* and energy resolution is worse at the centre compared to the ends (as shown in Figure 4.3).

In a single pixel detector the thermalisation time is typically much shorter than the effective TES-bath time constant and ΔE is independent of thermal conduction to the bath *G* (adjusting *G* will affect the signal and the noise equally and will only change the decay time constant). In the case of the DROID,

it is the thermalisation time in the absorber that provides the position discrimination, so it must not be too small. Scaling both G and G_{abs} by the same amount (recalling from Table 3.1 that G_{abs} is the thermal conductance of the DROID absorber), changes the frequency spectrum of both the signal and the noise but the *NEP* and hence the energy resolution remains unchanged, whereas adjusting the ratio $g = G_{abs}/G$, will affect the resolution. Thus the g ratio is a critical design parameter which can be used to optimise the detector design depending upon the required application. In practical terms, g may be adjusted by changing device material and geometry (length, width, thickness); though this may also have an effect on other device parameters such as wide field coverage and detection efficiency.



Figure 4.5. Energy resolution ΔE across DROID 1 using the optimised filtering method for g values increasing in one order of magnitude steps between g = 0.001 (top line) and g = 1000 (bottom line).

Figure 4.5 shows ΔE for DROID 1 using the optimised filtering, for different values of the conductance ratio $g = G_{abs}/G$. In the case of a sufficiently high g, the absorber thermalisation frequency is much higher than the signal bandwidth. We essentially have two TESs sharing one single absorber and our energy resolution across the complete detector, is the same as that expected from a single pixel detector, with the total heat capacity C_{tot} equal to the sum of both TESs capacities 2*C* and

absorber capacities C_{abs} ($C_{tot} = C_{abs} + 2C$). We will see how this affects position resolution in Section 4.4.2. Conversely, decreasing g degrades ΔE at the centre of the device (but actually improves ΔE slightly for absorption close to the TES itself). A sufficiently small g corresponds to a thermalisation time long compared to the effective TES time constant. The small DROID conductance attenuates the high frequency components of the signal, acting as a low pass filter. The effect on the noise is complicated by the frequency-dependent correlations between the two outputs when the signals are summed. However, the absorber phonon noise will be filtered in the same way as for the signal but the white noise level for TES Johnson noise and bath phonon noise will remain largely unaffected. Consequently the overall *SNR*, and therefore energy resolution, is reduced. The current DROID 1 has $g \approx 0.85$, which gives an energy resolution at the centre of the detector about a factor of two less than for the high g case.

4.4.2. Position Resolution

Knowing that ΔE varies with the conductance ratio $g = G_{abs}/G$, we can see that Δx must be similarly dependent upon the ratio g. Figure 4.6 shows Δx across DROID 1 for different g values (with E = 5.9keV) using the energy ratio method and Figure 4.7 using the optimised filtering method. Both methods show the largest spatial variation in Δx for low g values. For low g, Δx found from the energy ratio method is heavily degraded at the ends where the energy ratio sensitivity (S_F) is lowest. Thus, we see that the assumption of Equation 4.19, that the energy and position resolving powers are of the same order at the centre of the device, is an inadequate approximation, given that Δx could be several orders of magnitude worse at the ends than at the centre of the DROID. Conversely, the optimum method makes use of the complete variation in pulse shape with position (S), not just the energy, therefore at the ends of the absorber it is the pulse shape which provides most of the position information while at the centre it is the amplitude (where the difference between the two methods is much smaller). With a large absorber conductance (large g) both methods show much less variation with position, but the optimum filter gives a much higher absolute resolution. In the limit as g becomes very large, both signals tend towards the same form and there is no information from which to derive the position, hence $\Delta x \rightarrow L$. Figure 4.8 compares the two methods for the experimental DROID 1. The g value for this device is close to the optimum value using the optimum filter but is far from optimum for the energy ratio method. Noting that $\Delta x \propto 1/E$, the position resolution requirements for XEUS (< 250 μ m) will only be met for the complete energy range (~ 1-12 keV) for the newly developed filtering method. The significance of this optimisation and impact on the science return and device design is clear.



Figure 4.6. Position resolution Δx along the length of DROID 1 using the energy ratio method, for g values from 0.001, increasing in factors of 10 to 1000. The X-ray energy is 5.9 keV.



Figure 4.7. Spatial resolution along the length of DROID 1 using the optimised filtering method, for g values from 0.001, increasing in factors of 10 to 1000. The X-ray energy is 5.9 keV.



Figure 4.8. Spatial resolution along the length of DROID 1 assuming E = 5.9 keV. Both the energy ratio method (*) and the optimised filtering method (\Box) are included. Also shown are the individual contributions to the energy ratio method from the covariance (-) and the contributions from each TES (\diamond); the quadrature sum of these terms gives the total.

4.4.3. Position and Energy Resolution Trade Off

Figure 4.9 shows the limiting Δx (i.e. the worst Δx along the length of the DROID) as a function of *g*, for both methods as well as the limiting ΔE (which is the same for both methods). The best achievable position resolution occurs at different *g* values for the different methods, corresponding to different ΔE . Consequently, the position determination method impacts directly on the device design. The energy ratio method gives the best position resolution at $g \approx 30$, whereas for the optimum filtering method it occurs at $g \approx 1$. For the experimental DROID 1, with $g \approx 0.85$, we find ΔE is a factor ~ 2.3 from the best achievable and using the new filtering algorithms, the position resolution is close to optimum, with a resolution of 1 part in ~ 1500 at 5.9 keV, whereas using the energy ratio method, Δx is a factor ~ 40 from the best achievable. Increasing *g* will improve ΔE at the expense of Δx . Consequently these must be traded in future DROID designs depending upon application.



Figure 4.9. Δx for the energy ratio method (solid) and optimised filtering method (dot dash) of determining the event position for DROID 1 as a function of g. Also shown is the limiting ΔE across the DROID (dashed), corresponding to the right axis. E = 5.9 keV

The other parameters affecting the detector time constants are the thermal capacities. If both the absorber heat capacity C_{abs} and TES heat capacity C are reduced, while keeping the ratio $c = C_{abs}/C$ the same, the energy and position resolutions will be improved (by the square root of the capacities) without changing the shape of the curves in Figure 4.9. C is fixed by the saturation conditions i.e. the requirement for temperature excursions not to drive the TES completely normal, whereas C_{abs} can be varied by absorber geometry and material choice. The effect of changing C_{abs} while keeping C constant is shown in Figure 4.10, where we reduce the absorber heat capacity by a factor 10. There is an overall improvement in resolutions due to the reduction in the total heat capacity but the shapes of the curves have also changed so that the optimum position resolution occurs for $g \approx 0.1$.



Figure 4.10. As for Figure 4.9, except $C_{abs} = C_{abs}/10$.

4.4.4. Optimisation of Design for High Resolution with Maximum Spatial Coverage

The original design aim of the DROID concept is to increase the number of effective pixels for a fixed number of read-out channels. This is achieved by increasing the length of the absorber, which in turn increases the heat capacity and decreases the thermal conductance and will consequently affect both the energy and position resolutions. The analysis above can be extended to illustrate the dependence of the energy and position resolution on the absorber length. Figure 4.11 shows ΔE and Δx (for two photon energies) as a function of the DROID absorber length. For this analysis we use a device with optimised TESs (with sensitivity $\alpha = 90$ and transition temperature $T_C = 120$ mK). In the first instance we set G = 2 nW K⁻¹. The absorber thickness and material are the same as for DROID 1.



Figure 4.11. Energy resolution (dashed line, right hand axis) and the position resolution (solid line, left hand axis) versus DROID absorber length. The position resolution is shown for E = 0.1 keV (top line) and E = 6 keV (bottom line). G = 2 nWK⁻¹.

Noting that $\Delta x \propto 1/E$, improvements in Δx for E > 0.1 keV are possible compared to pixel arrays, where Δx is limited by pixel pitch (~ 250 µm) and is independent of *E*. For large absorber lengths the resolution is degraded because the *g* ratio is low and the total heat capacity is high. To ensure a high *g* ratio over all absorber lengths, as is required for good energy resolution performance, we reduce the TES-bath conductance. Figure 4.12 again shows ΔE and Δx but now we set G = 0.2 nWK⁻¹. We see this has improved ΔE by approximately a factor of two for L = 8 mm with no significant change in Δx . Further optimisations should be possible by absorber material selection which will intrinsically change C_{abs} and G_{abs} (see Chapter 6 for a discussion of absorber material selection).



Figure 4.12. As Figure 4.11, except $G = 0.2 \ nWK^{-1}$.

4.5. Practical Implementation

The analysis methods described here assume that the pulse shapes and hence the filters, are known, this in turn implies that the position is known. Consequently, an iterative approach is needed to find the event position. A method is required which gives an initial estimate of the position without *a priori* assumptions. This non-optimum estimate is then used to determine the correct filter to use, possibly by interpolating between a set of stored templates which have been pre-computed for increments in x, either from simulated or prior calibration data. If the new estimate of x is significantly different than the original one, a further iteration may be required. This procedure could be computationally intensive and work is ongoing to develop efficient algorithms.

4.6. Summary

In this Chapter we have derived expressions for the energy resolution of the twin TES system, using improved digital filters for optimised performance. We have shown that the technique of adding the

two TES signals and filtering the result to obtain an energy estimate, does not return optimum results under certain circumstances, specifically in cases of a sufficiently low g ratio. Further, the DROID detector requires that a filter be applied to each TES signal for the determination of the event position, so different filters are required in any case. We have shown that using these improved optimal filters, modest theoretical improvements in ΔE can be made for the case of $|S_1(f) - S_2(f)| >> 0$ i.e. at the device ends, for sufficiently small g, though the limiting resolution (at x = L/2 for a symmetric detector) will not be improved. Thus the detector must still be designed with this limiting resolution in mind. The extension of the optimal filtering theory into the position determination has shown impressive gains in Δx (over all g space) over the standard method of using the normalised energy ratio of the two TES signals. This analysis provides, for the first time, a complete theoretical framework of the optimised design of DROIDs for specific applications such as XEUS, or more recently, X-ray interferometry [Willingale, 2004], where there is a need for $\Delta x \sim 10{\text{-}}30 \ \mu\text{m}$ (in one-dimension) coupled with $\Delta E <$ 100 eV over the soft X-ray band (0.5-2 keV). Potentially the analysis developed within could make these goals realistically achievable for an optimised DROID. In general, the requirements for the optimisation of a single pixel TES hold true for a DROID as well (low total heat capacity, high TES sensitivity and rapid absorber thermalisation). We have largely focused on the optimisation of Δx and ΔE via the adjustment of c and g but it is also important to consider that these parameters will impact directly on detection efficiency and count rate, which in turn affect the science goals. We should also note that the theory described above does not consider practical factors (material choice and geometry, thermal engineering for example) which may limit the actual workable parameter space.

The filtering algorithms developed within may also be applicable to other position-sensitive technologies such as Superconducting Tunnel Junction (STJ) DROIDs [den Hartog et al., 2002; Verhoeve et al., 2000]. Here, excited quasi-particles generated by photon absorption in a superconducting absorber, diffuse spatially, generating a charge pulse in the two STJs. The signals are typically read out using charge sensitive preamplifiers with an analogue shaping stage. Position information is then determined using normalised energy ratios. The optimised filtering techniques we have developed for TES DROIDs can be applied to the STJ DROID data in the same way. Thus improved position resolution in these detectors can also be expected.

Chapter 5: Read-Out and Cooling Requirements

In this Chapter we describe the equipment and techniques required for operating TES detectors in the \sim mK temperature régime. This will include an outline of the Leicester cryogenic test facilities; in particular, a discussion of the operation of Dilution Refrigerators (DRs). We begin by examining the electronic read-out requirements for TES detectors.

5.1. SQUID Read-Out of TES Detectors

5.1.1. SQUID Principles

The standard read-out for TES devices is the Superconducting QUantum Interference Device or SQUID. Conventional semiconductor amplifiers such as Silicon JFETS (with typically ~ 100 k Ω impedance) are used to read-out CCD and STJ detectors but are not well impedance matched to ~ m Ω TESs. SQUIDs have lower intrinsic noise, lower power dissipation (typically ~ μ W compared to ~ mW for JFETs), can operate at mK temperatures (and so can be mounted close to the detector) and are more closely matched to the impedance of the TES. SQUIDs are capable of detecting very small magnetic fields, typically < 10⁻¹⁴ T; for comparison, the Earth's field is ~ 10⁻⁶ T, the field created by the human heart is ~ 10⁻¹⁰ T and by the brain ~ 10⁻¹³ T. SQUIDs are highly sensitive magnetometers that can be used for a variety of applications. In the present application the SQUID is inductively coupled to a TES via an input coil with a typical inductance L_{in} ~ 100 nH. The absorption of an X-ray photon in the TES causes a current pulse in the input coil and therefore a change in magnetic flux Φ applied to the SQUID. This change in flux causes a finite voltage change across the SQUID, which is then amplified and integrated by room temperature electronics and can be displayed on an oscilloscope or logged via an analogue-to-digital converter (ADC). Figure 5.1 shows the TES bias and SQUID read-out circuitry.

There are two different types of SQUID, AC (or RF) SQUIDs and DC SQUIDs [Swithenby, 1980]. DC SQUIDs are generally considered easier to analyse than their AC equivalents and are the type used

to read-out TES detectors. The DC (direct current) SQUID is essentially a flux-to-voltage transducer based on two Josephson junctions in a superconducting ring; the AC (alternating current) SQUID is based on a single junction only.



Figure 5.1. Bias and read-out circuitry for a TES detector. Read-out is via a SQUID with preamplifier and integrator, also shown is the feedback circuitry for operation in Flux Lock Loop (FLL) mode (see Section 5.1.2). Dual channel read-out of a DROID detector requires two identical bias circuits and SQUIDs with dual amplifier chain.

In addition to "conventional" SQUID read-out, it has been theoretically demonstrated that a transformer based read-out could also be used to read out microcalorimeter devices [Sushkov, 2004]. In this scenario, the TES is inductively coupled to a warm front-end amplifier via a transformer coil. The TES can be AC biased through the same read-out circuitry, so there is no need for a cold bias circuit. This technique may open up new possibilities in detector array design; making biasing simpler and reducing the thermal loading at the cold detector plane while maintaining good energy resolution (\sim eV). In this method there is no need for the use of SQUIDs at all, thus making it a less expensive and technically simpler alternative. This technique requires further evaluation to see whether it can be used to read-out DROID devices.

The DC SQUIDs used in this thesis were made by Oxford Instruments Superconductivity (OIS) [Polushkin et al. 2002]. These SQUIDs use an intermediary transformer between the input coil and the SQUID in order to increase the mutual inductance, and hence the gain, between the SQUID and input coil. The alternative to using an intermediary transformer based SQUID is to use a series array of single SQUIDs. Such SQUID arrays can consist of \sim 100 DC SQUIDs in series, which act to provide a much larger signal than with a single SQUID alone [Welty and Martinis, 1990].

A single Josephson junction is made up of two superconductors, separated by a thin nonsuperconducting layer – of thickness typically of the order of the coherence length (\sim nm). This weak link allows superconducting electrons (Cooper pairs) to tunnel through with no resistance [Gallop, 1991]. A DC SQUID consists of a superconducting ring interrupted by two such identical Josephson junctions (see Figure 5.2). A fundamental property of superconducting rings is that they can enclose magnetic flux only in multiples of a universal constant called the flux quantum, $\Phi_0 = h/2e = 2.07 \times 10^{-15}$ Wb (Vs). This arises because the Cooper pairs share a common wavefunction which must be single valued at any point in the ring, with a phase change around the closed loop of $\pm 2\pi n$ where *n* is a positive integer [Hook and Hall, 1999].



Figure 5.2. Schematic diagram of a DC SQUID showing the magnetic flux through the superconducting ring.

In zero applied magnetic field the current needed to drive the SQUID into the normal state, critical current, is twice the critical current of each junction $(2I_C)$. When a bias current I_{sq} applied to the superconducting ring exceeds the critical current I_C of the junctions; a finite voltage difference across the junctions is produced. Figure 5.3 shows the experimentally obtained I-V curve of a typical SQUID used in this report; here the bias current is swept symmetrically about zero and the output voltage V_{out} measured. The resistive branches appear when the bias current exceeds the critical current of the SQUID. In order to maximise V_{out} , the bias current is set so that the SQUID is most sensitive to changes in applied magnetic field. This occurs at the 'knee' of the SQUID I-V curve, as shown in Figure 5.3, between the superconducting and normal state of the junctions where $I_{sq} \sim 2I_C$. Changes in the applied magnetic field induce a screening current in the SQUID ring. This screening current acts to oppose the applied magnetic field maintaining flux quantisation in the ring. The screening current superimposes on the bias current and results in a modulation of the critical currents of the two junctions. The critical current of the SQUID varies periodically with the applied flux, with a period corresponding to one flux quantum, Φ_0 . The maximum in the critical current occurs when the applied flux is equal to an integer value of the flux quantum ($n\Phi_0$) and the minimum in critical current occurs at half integer values of the flux quantum $(n + \frac{1}{2}\Phi_0)$. This periodicity arises because at half integer

values of Φ_0 , it becomes energetically more favourable to allow the penetration of an additional flux quantum into the SQUID ring, rather than to keep the flux out; consequently the screening current and therefore the critical current of the SQUID are periodic with the applied flux. The modulation in the critical current in turn causes a modulation of V_{out} , which is also periodic with the applied flux. This effect is demonstrated in Figure 5.4 where the quasi-sinusoidal output voltage V_{out} is shown as a function of applied magnetic flux Φ_{in} (by modulating the Φ_{bias} as shown in Figure 5.1 using a triangle wave input) – this constitutes a macroscopic observation of a quantum mechanical effect.



Figure 5.3. I-V curve for an Oxford Instruments SQUID. The bias current I_{sq} is driven symmetrically around $I_{sq} = 0$. The flat region in the centre of the diagram is where the junctions are completely superconducting. As the critical current of the junctions I_C is exceeded, the normal branches appear. The SQUID is most sensitive to changes in applied flux when I_{sq} is set to $\sim \pm 2I_C$. The experimental SQUID data shown in this Figure and throughout this Chapter were taken by the present author and Dr. David Goldie at the Cavendish Laboratory, University of Cambridge.



Figure 5.4. *V*- Φ curve for an Oxford Instruments SQUID. This Figure shows the output voltage variation for a steadily increasing magnetic flux through the SQUID ring. The flux is varied by driving a current I_{fbk} through the feedback coil through the Φ_{bias} input (see Figure 5.1). One complete current cycle corresponds to a single flux quantum Φ_0 . The output voltage modulation depth ΔV is at a maximum when the current bias I_{sq} is set to the sensitive 'knee' of the I-V curve (~ 2I_C as shown in Figure 5.3).

5.1.2. Operation in Flux Lock Loop Mode

SQUIDs are usually operated in flux-locked-loop (FLL) mode. Here the SQUID response is linearised by applying negative feedback to the SQUID ring [Drung, 2003]. In FLL mode the signal output is amplified, integrated and fed-back into the SQUID via a feedback resistor R_f (typically $R_f \sim k\Omega$) and an inductively coupled feedback coil with mutual inductance M_f . This closed loop feedback circuit, as shown in Figure 5.1, acts to cancel the applied field and therefore keeps a near zero, but constant flux through the SQUID. The current flowing through the feedback coil is then proportional to the current through the input coil. Consequently, the output voltage V_{out} must also be directly proportional to the current through the input coil and therefore independent on the actual shape of the basic $V-\Phi$ curve. This is advantageous since the linear flux range $\partial \Phi$ of the $V-\Phi$ curve is small, therefore any X-rays which caused a change in applied field exceeding $\pm \partial \Phi/2$, would result in a non-linear response. In FFL mode the output voltage and the input flux are dependent only upon the feedback parameters R_f and M_f :

$$V_{out} = \frac{R_f}{M_f} \Phi_{in}$$
$$= R_f \frac{M_{in}}{M_f} I_{in},$$
 5.1

where I_{in} is the input coil current.



Figure 5.5. Close-up view of V- Φ curve at the zero-point crossover where the gradient $dV/d\Phi_0$ is a maximum.

Maximum signal-to-noise is achieved when the feedback bias is chosen to give the biggest flux-tovoltage transfer function $dV/d\Phi_0$. Therefore the feedback bias is set to the maximum gradient of the $V-\Phi$ curve (as indicated on Figure 5.5) before the FLL mode is activated. The linearised closed loop response in FLL mode is shown in Figure 5.6



Figure 5.6. SQUID V- Φ curve in FLL mode. The SQUID output is now a linear function of the current in the feedback coil and hence the applied magnetic field.

Table 5.1 summarises the experimentally determined parameters for the two different SQUIDs used in this report.

Table 5.1. Experimental SQUID parameters for two Oxford Instruments SQUIDs.

	SQUID 1	SQUID 2	
I_C - Critical current	28 µA	24 μΑ	
ΔV - Modulation depth	50 µV	68 µV	
$dV/d\Phi_0$ - Transfer function	$120 \ \mu V / \Phi_0$	187 $\mu V/\Phi_0$	
M_{in} - Mutual inductance of input	625 pH	681 pH	
M_f - Mutual inductance of feedback	25 pH	25 pH	

The most important parameters of a SQUID are its noise and dynamic behaviour (bandwidth and slew rate). Generally, the intrinsic flux noise of the SQUID and the amplifier noise determine the overall noise performance of the SQUID read-out; the FLL electronics determine the dynamic behaviour.

In a SQUID read-out system the total noise referred to the input will be governed by noise from the preamplifier and the intrinsic noise from the SQUID. Ideally the noise from the SQUID and amplifier will be much less than the detector noise and so will not contribute to degradation of the energy resolution.

5.1.3.1. SQUID Noise

In a practical SQUID system, shunt resistors (~ 1 Ω) are used in parallel with the Josephson junctions to remove hysteresis in the *I-V* curve [Gallop, 1991]. The Johnson noise of these shunt resistors determines the intrinsic SQUID noise. This Johnson noise can be expressed as an equivalent flux noise Φ_n in the SQUID ring, which may be approximated to [Frericks et al., 1998]:

$$\Phi_n = \left(\frac{16k_B T L_{sq}^2}{R_{sh}}\right)^{\frac{1}{2}},$$
5.2

where k_b is Boltzmann's constant, *T* is the operating temperature, R_{sh} is the shunt resistance (~ 5 Ω) and L_{sq} is the SQUID inductance (~ 30 pH). This flux noise can be readily converted to an equivalent current noise on the input coil i_n , via the mutual inductance M_{in} , which couples the input coil to the SQUID ring, $i_n = \Phi_n/M_{in}$. Our SQUIDs are mounted at the same stage of the cryogenic cooler as the detectors and therefore operate in the temperature régime $T \sim 10\text{-}200$ mK. At these temperatures the current noise from the SQUID will be negligible (< 0.1 pA/ $\sqrt{\text{Hz}}$) and the noise from the preamplifier dominates.

5.1.3.2. Amplifier noise

An amplifier with shorted input will exhibit two sources of noise; (i) a voltage noise V_n and (ii) a current noise I_n (which flows from the amplifier input). An amplifier connected to a SQUID will see a dynamic resistance $R_{dym} = \partial V/\partial I$ [Drung, 2003]. Thus, the total amplifier voltage noise will be the sum of the noise powers from these two contributions:

$$V_{amp} = \sqrt{(I_n R_{dym})^2 + V_n^2} .$$
 5.3

The actual values of V_n and I_n will depend on the components used to manufacture the amplifier. A DC SQUID usually has a low dynamic resistance of order 1 Ω , and the effect of the current noise is negligible – therefore the voltage noise will dominate.

It is useful to consider the noise performance of an amplifier in terms of its noise temperature T_N . The noise temperature is defined by assuming that the amplifier is connected to a source resistor R_S at temperature T_S with an associated Johnson noise $V_{ns} = (4k_b T_S R_S)^{1/2}$ [Drung, 2003]. The total voltage spectral density including amplifier current and voltage noise, as well as the SQUID Johnson noise, is then:

$$V_{tot} = \sqrt{(I_n R_{dyn})^2 + V_n^2 + V_{ns}^2} , \qquad 5.4$$

or, substituting for V_{ns} :

$$V_{tot}^{2} = (I_{n}R_{dyn})^{2} + V_{n}^{2} + 4k_{b}T_{S}R_{S}.$$
5.5

It is then possible to express the amplifier noise in terms of an equivalent temperature T_N . By setting $R_{dyn} = R_S$, the total noise becomes:

$$V_{tot}^2 = 4k_b (T_N + T_S) R_S \,.$$
 5.6

Equating Equations 5.5 and 5.6 and rearranging, the amplifier noise temperature T_N is then:

$$T_{N} = \frac{1}{4k_{b}} \left(I_{n}^{2} R_{S} + \frac{V_{n}^{2}}{R_{S}} \right).$$
 5.7

Thus, the effect of the amplifier noise can be explained by an increased temperature T_N above the operating temperature T_S . The minimum achievable noise temperature is then associated with an ideal source resistance R_S . Differentiating Equation 5.7 with respect to R_S and setting the result to zero gives:

$$\frac{dT_N}{dR_S} = \frac{1}{4k_b} \left(I_n^2 - \frac{V_n^2}{R_S^2} \right) = 0.$$
 5.8

The optimised source resistance is then $R_S = V_n/I_n$, where the contributions from the voltage noise V_n and current noise I_n are equal. So for optimised noise temperature we require an amplifier matched to

 R_S . This is not always possible, however, and as previously stated the individual noise contributions from the amplifier are determined by the transistor components used in its design. The preamplifier matching to the source resistance R_S can be improved by connecting N transistors in parallel [Polushkin, 2004] (although practically N will be limited by the bandwidth requirements, since increasing N will also increase the capacitance [Ashton, 2004]). In this situation V_n^2 decreases linearly with N and I_n^2 increases linearly with N, thus R_S will scale as 1/N. Changing N will therefore change R_S but does not change the minimum achievable noise temperature, which is dependent primarily upon the transistor type.

5.1.4. Dynamic Behaviour: Slew Rate and Bandwidth

Apart from noise, the two key parameters for SQUID performance are the slew rate and bandwidth. The maximum slew rate $(d\Phi/dt)$ is determined by the maximum rate at which a signal can be tracked by the SQUID. The system slew rate is defined as [Drung, 2003]:

$$\left(\frac{\partial\Phi}{\partial t}\right)_{\max} = 2\pi B_{-3db} \partial\Phi , \qquad 5.9$$

where $\delta \Phi$ is the region of the *V*- Φ curve which is approximately linear (assuming a sinusoidal *V*- Φ curve, $\delta \Phi \approx \Phi_0/2\pi$) and $B_{-3db} = 2\pi\tau_{eff}$ is the -3db bandwidth of the combined SQUID and amplifier frequency response, here τ_{eff} is the effective time constant in FLL mode. The maximum current slew rate in the SQUID input coil is then:

$$\left(\frac{\partial i}{\partial t}\right)_{\max} = \frac{1}{M_{in}} \frac{\partial \Phi}{\partial t}_{\max}.$$
5.10

Thus, in contrast to the noise performance, the slew rate is actually degraded by an increased input inductance M_{in} , so there exists a trade-off between the two. The maximum slew rate produced by X-ray pulses in the input coil is determined by the intrinsic rise time and amplitude response of the detector. Typically the pulses have peak amplitudes of microamps associated with microsecond rise times, thus the slew rate requirement is for $\partial i/\partial t > 1$ A S⁻¹ and for a bandwidth of $B_{-3db} \ge 1$ MHz.

5.1.5. Comparison of Different SQUID Amplifier Performance

Two different amplifiers have been investigated. The first of which was supplied by Oxford Instruments. The second was supplied by the Cavendish Laboratory (Dr. David Goldie) and was a later version of the original Oxford Instruments design. Table 5.2 summarises the noise and dynamic behaviour of these two amplifiers.

Table 5.2. Comparison of preamplifier performance parameters for two different read-out chains. Read-Out 1 is defined as the non-optimum amplifier chain used in with SQUID 1 (see Table 5.1) and Read-Out 2 as the new transistor based amplifier chain used with SQUID 2 (see Table 5.1). Note that the equivalent input noise for Read-Out 1 is an estimated parameter whereas all other parameters are experimentally measured.

Parameter	Read-Out 1	Read-Out 2	
<i>i_n</i> - Equivalent Input Noise	22 pA/√Hz	6 pA/√Hz	
$\partial i/\partial t$ - Slew Rate	50 mAs ⁻¹	10 As ⁻¹	
B_{-3db} - Bandwidth	32 kHz	3 MHz	
$ au_{rise}$ - Rise Time Limit	10 µS	0.1 µS	

The initial, non-optimised, preamplifier used for testing (Read-Out 1 as listed in Table 5.2) was based around a bipolar operational amplifier Analogue Devices AD 797 with an intrinsic voltage noise of V_n ~ 0.9 nV/ $\sqrt{\text{Hz}}$ and a white current noise I_n of a few pA/ $\sqrt{\text{Hz}}$. Using Equation 5.4 to estimate the total amplifier noise V_{amp} and converting this to an equivalent current noise at the input, we have:

$$i_n = \frac{\Phi_0 V_{amp}}{M_{in}} \frac{\partial \Phi_0}{\partial V}$$

 $\sim 22 \text{ pA}/\sqrt{\text{Hz}}.$

This level of noise is comparable to the calculated intrinsic current noise expected from the DROID (see Figures 3.11 and 3.12) and will consequently have a degrading effect on the detector energy resolution.

The bandwidth and slew rate of the amplifier used for the initial experiments are measured to be, $B_{-3db} \sim 32$ kHz and $\partial i/\partial t \sim 50$ mA s⁻¹ respectively. This again severely limits the overall performance of our system and was further illustrated by the constant loss of flux lock when large X-ray pulses occurred. This was particularly troublesome when biasing at the low end of the transition where the TES resistance is low and the bias current high (so that the slew rate is higher). From Equation 5.9, high bandwidth is required for high slew rate but in addition, if the signal band is outside the read-out bandwidth then the high frequency components in the signal will be lost. The high frequency components in the signal define the rise time of the pulse. Consequently, a restricted bandwidth can limit the maximum rise time of the pulses and reduce the peak pulse height on the output. Assuming a one-pole (RC) filter roll-off in the frequency domain, the bandwidth and rise time (taken from the 10-90 % limits) are related by [Hamilton, 2003]:

$$\tau_{rise} \approx \frac{0.35}{B_{-3db}},$$
5.11

which, in our case means the pulse rise times at the output will be limited to $\geq 10 \ \mu$ s, consistent with experimental data. This is not ideal for the DROID detector where we rely on the high frequency components in the signal to determine the event position. Returning to Figure 3.8, we see that for our current prototype DROID 1 < 15 % of the total number of pulses have rise times faster than this and will be affected. For these pulses, at the ends of the DROID, the position resolution may be degraded. It is clear that for both dynamic behaviour conditions the initial amplifier performance was approximately two orders of magnitude worse than required. The noise performance can be improved by using a lower noise transistor based amplifier, or by using a more highly optimised SQUID with a larger transfer function $\partial V/\partial \Phi_0$ or a larger input inductance M_{in} . Subsequent testing has used a transistor based preamplifier with smaller intrinsic voltage noise supplied by the Cavendish Laboratory in Cambridge. Experimental testing using this amplifier has demonstrated an equivalent input current noise of ~ 6 pA/ \sqrt{Hz} . This second preamplifier chain provided by the Cavendish also improved the bandwidth and slew rate to ~ 3 MHz and ~ 10 As⁻¹ respectively, meaning that both the noise and dynamic behaviour more than match the detector requirements and will not limit spectroscopic or imaging performance.

5.2. Data Acquisition

Data acquisition is in the form of an in-house built (Mr. Terry Ashton) PCI card with dual 14-bit ADC, with on board digital signal processor (DSP) and associated software written (by the present author) in VisualBasic. The output signal from the amplifier chain is connected to the ADC in a PC via a coaxial cable. Data can be recorded from either a single channel or from dual channels simultaneously; this facilitates data acquisition from two TESs in a DROID or from a single TES. The software allows data to be obtained in a continuous stream at variable sampling rates from 1 kHz to 416 kHz. This is ideal for noise analysis where large sections of baseline data are required without any X-ray contamination. This mode was also used for continuous tracing of the TES *I-V* curves. A second mode is also available for triggering on the edge of a pulse on arrival. 4096 samples at 500 KHz for both single and dual channel modes are recorded per event and written to file with the digital signal filtering applied post-facto. The data is written to file in binary format with two bytes per sample.

5.3. Operation at Ultra Low Temperatures

5.3.1. Operating Principles of the ³He/⁴He Dilution Refrigerator

The He³/He⁴ Dilution Refrigerator (DR) is the most commonly used system for ~ mK operation in a laboratory environment, offering cooling powers of typically 100 μ W at 100 mK and base temperatures of ~ 10 mK. The process which allows a DR to cool a detector and SQUID assembly from room temperature to base temperature is complex and involves several stages.

A standard DR consists of an experimental insert, which is thermally isolated in an inner vacuum chamber (IVC). The experimentation is mounted on the base of the insert on a part of the DR called the mixing chamber (MC); it is in the MC that the final stage cooling takes place. Both the TES and SQUID are very sensitive to stray magnetic fields. Thus, one or more magnetic shields are often used to enclose the experimental area. We use an outer Mu metal (nickel-iron alloy) shield over the IVC can. Mu metal is a high-permeability, magnetically "soft" alloy ideal for magnetic shielding against mT stray fields. In addition to the mu-metal shield we also use a smaller superconducting lead shield $(T_c = 7.2 \text{ K})$ inside the IVC can which encapsulates the entire experimental volume. In this case the superconducting lead will act to expel any stray fields. The SQUID itself is housed in a small Niobium can ($T_c = 9.3$ K) for added shielding (see Figure 5.9). The entire insert is contained within a Dewar of liquid Helium (LHe) at 4.2 K (main bath). Because of the high cost of LHe, liquid Nitrogen (LN) is often used to the pre-cool the dewar to 77 K, at which point the Nitrogen may be blown out (using room temperature He gas) and replaced with LHe to cool the insert to 4.2 K. An outer vacuum chamber (OVC) provides the final thermal isolation from room temperature. To achieve ~ mK base temperatures the DR relies on the circulation of two isotopes of Helium: He³ and He⁴, through the experimental insert [Balshaw, 2001]. Initially, this mixture is condensed into the circulating path through firstly the LHe main bath and then a small pot of pumped LHe (1 K pot). The LHe for the 1 K pot is picked up from the main bath, and pumped on to pre-cool the warm incoming mixture to ~ 1.5 K. Once the circulating mixture has been condensed into the insert, a room temperature pumping system is used to slowly circulate the mixture. At the start of the circulation path, before entry to the insert, the mixture is passed through a filter (LN cold trap) which acts to freeze out any contamination (typically water or oxygen) that could cause a blockage further down in the system.



Figure 5.7. Phase diagram of ³*He*/⁴*He mixtures [Balshaw, 2001].*



Figure 5.8. Schematic of the mixing chamber of a typical dilution refrigerator where phase separation takes place [Balshaw, 2001].

Figure 5.9. Detector and SQUID mounted on the mixing chamber of a dilution refrigerator at Leicester.

When a mixture of the two Helium isotopes is cooled below 0.86 K (the tri-critical point), it separates into two phases [Griffiths, 1970]: 'dilute' and 'concentrated' (see Figure 5.7 for a phase diagram of the ${}^{3}\text{He}/{}^{4}\text{He}$ mixture). The so-called concentrated phase is rich in ${}^{3}\text{He}$ and is less dense than the dilute

phase, which is rich in ⁴He. The enthalpy of the ³He in the two phases is different and so it is possible to obtain a cooling effect by what can be thought of as 'evaporating' the ³He from the concentrated phase into the dilute phase (ultimately this is a process described by Quantum Mechanics and beyond the scope of this report). This evaporation-like process provides the final cooling power from 0.86 K to base temperature and occurs at the phase boundary in the mixing chamber, onto which the detector assembly is attached (see Figures 5.8 and 5.9).

DRs can however be difficult to operate, and experimental delays can occur due to leaks in the vacuum chambers or to blockages in the circulating path (usually due to ice), which can require the whole system to be warmed up again and pumped out.

At the University of Leicester we have had access to two different DRs for experimental testing.

5.3.2. Side-Arm Dilution Refrigerator

Our main DR is a semi-custom design from Oxford Instruments Superconductivity (OIS) (Tubney Woods, Abingdon, Oxfordshire, OX13 5QX). This DR, shown in Figures 5.10 and 5.11 has an external side-arm to allow for the attachment of an X-ray beamline. This is ideal for prolonged X-ray testing of devices with a variety of energies using an external source. However, the side-arm DR has the disadvantage of more complex access to the mixing chamber than for a standard DR (see Section 5.3.3). In contrast with the more conventional top-loading DRs, where the insert may be cleanly lifted out of the main bath, the only access to the mixing chamber in our DR is through the lower section – thus the OVC seal must be broken. This means the turn-round time can be greater than one week. The entire system needs to be warmed to room temperature and the OVC re-pumped after reassembly. The side-arm DR also has an outer jacket of LN, which acts to shield the LHe main bath from thermal radiation. This DR is operated in a 0.5 m deep pit to enable easy access to all operational components and the circulating pump will be situated in an isolated room with the bellows run through a sand-pit. This installation will be carried-out in the near future and aims to limit the transmission of vibrations to the DR, which can cause additional noise due to microphonics. Due to a prolonged period of refurbishment, this DR has not been used for the work reported in this thesis. However, it is now operational and all future TES testing at Leicester will be carried out using this system.



Figure 5.10. Photograph of the side-arm Dilution Refrigerator.



Figure 5.11. Schematic diagram of the lower section of the side-arm DR [Trowell, 2001].

5.3.3. Top-Loading Dilution Refrigerator

A standard 'off the shelf' DR was provided on loan to Leicester by OIS over the period 2002 - 2004. This has a standard single LHe main bath (with no LN outer jacket) and unlike the side-arm DR, has a top-loading insert. This means that the key OVC seals do not have to be broken to change the experimental insert. This DR has provided faster experimental turn-around times than the side-arm fridge. This DR does not however have a side-arm for external beamline attachment, thus X-ray testing requires the source to be mounted in the fridge with no shuttering capability. This DR also has a slightly smaller IVC chamber thus limiting the experimental payload capacity.

The standard wiring configuration in the top-loading fridge consists of 36 twisted pairs of Constantan wire in three looms. Constantan is copper/nickel alloy which has a very low thermal conductivity and very small temperature coefficient (~ $20 \,\mu K^{-1}$) of resistance. This is ideal for cryogenic systems where heat transported down the wiring can cause an excess heat load and adversely affect fridge performance. Constantan also has a relatively high resistivity with ~ 160 Ω per length of wire measured from the room temperature connectors to the mixing chamber of the DR. Unfortunately, Constantan is not ideal for SQUID operations and testing in the top-loading fridge revealed significant levels of excess noise. This was tracked primarily to Johnson noise from the high impedance wiring. The SQUID current bias and feedback bias requires low impedance wiring of a few ohms, thus additional low impedance wiring had to be installed. In addition to the noise level it was noticed that the fridge base temperature was dramatically affected by the application of the feedback bias. This base temperature drift meant that experiments could only be carried out for a few minutes before the system had to be left to recover. This was again tracked to a problem with the wiring and simple analysis revealed an excessive level of power dissipation in the high impedance wiring: $\sim 300 \ \mu W$ along the length of the fridge with ~ 7 μ W at the mixing chamber. Copper has a much higher resistivity temperature coefficient than the Constantan (~ 4.3 mK⁻¹) but also has a higher thermal conductivity. Thus, copper wiring was installed from the room temperature connectors to the 4.2 K stage of the fridge. Each wire had a measured resistance of ~ 4 Ω at room temperature, becoming negligible at 4.2 K. Copper could not be used from the 4.2 K to the mixing chamber because the high thermal conductivity would introduce heat loads. Superconducting wire intrinsically has a low thermal conductivity and is therefore ideal for low impedance, low conduction wiring as required here. Niobium/Titanium wire ($T_c \sim 10$ K) was installed from the 1 K stage to the mixing chamber of the fridge. This wiring had to be thermally anchored at each stage of the insert to insure that no heat was conducted to the mixing chamber. This wiring installation was carried out by the present author and Mr. R. Limpenny.



Figure 5.12. Photograph of the top-loading fridge insert with custom Cu + Nb/Ti wiring for SQUID operations.

Figure 5.12 shows the insert of the top-loading fridge with custom SQUID wring from the 4.2 K stage to the mixing chamber. Repeated SQUID testing in the fridge using this new low impedance wiring reduced the baseline noise by a factor of approximately three, comparable to levels observed in the Dewar testing. Furthermore, the anomalous heating due to the feedback bias was also successfully removed.

5.3.4. Adiabatic Demagnetisation Refrigerators

Another technology is widely used for the operation of TESs and other cryogenic detectors in the mK temperature régime. The Adiabatic Demagnetisation Refrigerator (ADR) is used as a laboratory cooler but importantly, is the only flight-qualified cooler capable of achieving the required operating temperatures [Bromiley, 1999; Porter et al., 2000a; Emes et al., 2002; Kushino et al., 2004]. This is in contrast to the DR described in the previous Section, where the circulation of the He mixture relies on gravity, thus making DRs unsuitable for the micro-gravity environment of space. The Wisconsin –

NASA GSFC collaboration successfully developed a single stage ADR for the X-ray Quantum Calorimeter (XQC) sounding rocket experiment, which flew in 1996 and 1999 [Porter et al., 2000a]. The ADR was mounted on a space-pumped LHe dewar and cooled the 36-pixel microcalorimeter array to a 60 mK base temperature with a hold time of > 12 hours. ADRs have not been used in the work reported in this thesis however the procurement of such a system is envisaged for the testing of TES arrays under development for the XEUS mission. A brief description of the operating principles will therefore be outlined here.

ADRs for use in future space missions will utilise mechanical cryocoolers to achieve a first stage cooling to temperatures of 6-10 K rather than the use of LHe (1.3-4.2 K), which would be impractical for long duration space missions. The final stage cooling is achieved using the demagnetisation of a paramagnetic salt pill (Ferric Ammonium Alum (FAA) for example), onto which the detector assembly is attached. This process is described by the laws of thermodynamics:

$$dU = TdS + BdM , \qquad 5.12$$

where T is the temperature, U is the internal energy, S is the entropy, M is the magnetisation and B is the applied field.

When a strong magnetic field (typically several Tesla) is applied to the pill, the magnetic dipoles will align with the external field, lowering the magnetic entropy of the system and generating heat, which is dumped into the thermal sink via a heat switch. After complete magnetisation of the sample, the switch is opened and the pill and experimental sample are thermally isolated from the external environment. The applied field is then adiabatically reduced so that the dipoles slowly relax, extracting entropy from the experimental sample (such that dS = 0), and decreasing the temperature of the whole system. By careful control of the applied field the temperature of the experimental attachment can be maintained constant. Once the pill has been completely demagnetised no more cooling is provided and the cycle has to be repeated. This recycling at regular intervals (~ 12 hrs) can take several hours and is an operational disadvantage compared to DRs, which can be operated continuously as long as the main bath remains filled with LHe. By using multistage ADRs, which essentially involve linking a series of ADRs in a chain, the heat sink temperature can be increased and the recycle time greatly reduced. Using this technique it is possible to operate the ADR continuously, where the final stage remains at a constant temperature. In a continuous ADR (CADR) [Shirron et al., 2004], the detector assembly stage is cooled periodically by a slightly colder stage, this is repeated in the other stages cascading heat up to the heat sink, thus by regulation of each stage the experimental stage can be used to provide continuous cooling and the base temperature is not compromised. Figure 5.13 shows a schematic diagram of a CADR.



Figure 5.13. Schematic diagram of a Continuous ADR [Shirron et al., 2004].

5.4. X-ray Sources

5.4.1. ⁵⁵Fe X-ray Source

To provide X-rays for the testing of the detectors reported in this thesis, a small ⁵⁵Fe X-ray (with activity typically of the order 100 kBq) source was attached to the mixing chamber of the DR, ~ 5-20 mm from the device (depending upon source activity and the required count rate), thus providing illumination of the entire detector area with X-ray photons. The aim was to achieve detector count rates of typically ~ 10 Hz, which in turn required fluxes at the detector plane of ~ 10^8 - 10^9 m⁻². ⁵⁵Fe is an unstable isotope of iron, with a half life of ~ 2.6 years, which decays via capture of an inner-shell (K) electron. The nucleus combines with the electron, turning a proton into a neutron and emitting a neutrino. Therefore the ⁵⁵Fe nucleus becomes ⁵⁵Mn and an outer electron fills the vacancy created in the K shell, emitting an X-ray photon in the process. The emitted X-rays are 90 % ⁵⁵Mn_{Ka} (E = 5895 eV) and 10 % ⁵⁵Mn_{Kβ} (E = 6490 eV).

5.4.2. X-ray Beamline Facility

Future X-ray testing using the side-arm DR will be carried using an external X-ray beamline facility. Using this facility, soft X-rays in the range ~ 0.3-3 keV are generated by the fluorescence emission of a coated copper anode stimulated by electron bombardment (from a heated tungsten cathode). A Bragg crystal monochromator is then used to separate the characteristic line emission from the background Bremsstrahlung continuum, produced by the acceleration of the charged particles. The crystals can be rotated to change the angle θ between the crystal lattice and the incident photons, thus the required wavelength can be selected subject to the Bragg condition ($n\lambda = 2d\sin(\theta)$). In order to cover the complete energy range, five different Bragg crystals are mounted on a wheel, which can be

selected as required. Table 5.3 summarises the typical energies produced by different coatings and the required Bragg crystal to select that energy.

Table 5.3. The main photon energies produced by the beamline facility for different anode coatings and Bragg crystal selection, also shown is the anode voltage required to generate them [Pearson, 2004].

Anode Coating	Crystal	Line emission	Energy (eV)	Voltage (V)
SiC	PbSt	Ск	277.0	750
CuO	PbSt	O _K	524.9	1000
MgF ₂	WtC	F _K	676.8	1250
CuO	RbAP	Cu _{La}	929.7	1750
NaCl	WtC	Na _{Kα}	1040.9	2000
MgF ₂	RbAP	Mg _{Kα}	1253.6	2500
Al ₂ O ₃	RbAP	$Al_{K\alpha}$	1486.7	2750
SiC	RbAP	$Si_{K\alpha 1}$	1740.0	3250
NaCl	ADP	Cl _{Ka1}	2622.3	5000
NaCl	ADP	$Cl_{K\beta}$	2815.6	5000

5.5. Summary

In this Chapter we have described in detail the operation of DC SQUIDs as required for the read-out of TES based detectors. We have reported on the characterisation of two different SQUID and read-out electronics chains. The first non-optimised read-out chain has been used in initial experimental studies and led to the development of the second chain, which is better optimised to meet the DROID requirements and will be used for future device testing. The operation of dilution refrigerators is also discussed in detail. All testing to date has been carried out in the top loading DR, though future testing will now migrate to the recently refurbished side arm DR in the purpose built cryogenics laboratory at Leicester.

Chapter 6: Detector Design and Material Characterisation

In this Chapter we discuss the key aspects of detector design. We start with a discussion of the physical requirements of a TES absorber and investigate the properties of metal, dielectric and superconducting absorbers, commonly used in microcalorimeters. We then discuss the design of the current prototype DROIDs, before describing the experimental characterisation of the electrical and thermal properties of the detector itself. This leads into a consideration of future optimisations of the DROIDs. Some of the results in this Chapter are published in [Smith et al., 2004; 2006c]

6.1. Absorber Material Selection

X-ray photons in the keV energy range interact primarily via the photoelectric effect [Jenkins et al., 1995]. Here, the primary X-ray is absorbed by an atom, which in turn ejects a bound inner shell electron, typically a M, L or K shell electron (though this will depend on the photon energy and the electron binding energies of the absorber material). The ejected photoelectron energy is the difference between the incident photon energy and the electron binding energies given off as either a characteristic X-ray photon, or through the emission of an Auger electron from an outer shell (if the photon is re-absorbed before escaping the atom). The probability of X-ray emission is described by the fluorescent yield (ω), which typically increases with atomic number (see for example Van Grieken and Markowic (2002) for tabulated data on fluorescent yields). The ejected photoelectrons and phonons. The deposited energy diffuses spatially and is detected by the thermometer as an increase in temperature.

The choice of absorber is driven by the requirement for high detection efficiency and a low heat capacity for optimum energy resolution. In a pixel detector rapid thermalisation is required for optimum resolution (Section 2.2.3), whereas in a DROID detector, we have seen in Chapter 4 that the absorber conductance G_{abs} is important for controlling thermal diffusion to the TES when optimising the energy and position resolution together.

The absorbed fraction of energy, equivalent to the quantum efficiency (QE), can be estimated from the Lambert-Beer law:

$$QE = 1 - e^{-\alpha \rho d}, \qquad 6.1$$

where *d* is the film thickness, ρ is the mass density and α is the mass absorption coefficient, taking into account photoelectric absorption and scattering, both coherent and incoherent. For soft X-rays, photoelectric absorption dominates and scales roughly as Z^4/E^3 [Krane, 1988]. Therefore high QE demands thick absorbers made from high Z elements. The problem with large volume absorbers is that they increase the total heat capacity of the detector and so may degrade the resolution (as per Equation 2.5) and slow the response time (as per Equation 2.14). Thus, absorber design and material selection are critical to the detector performance.

The total low temperature heat capacity C (J K⁻¹ m⁻³) of a material can be estimated from the Debye law [Hook and Hall, 1999]:

$$C = C_e + C_p$$
$$= \gamma T + \beta T^3.$$

where $C_e = \gamma T$ is the electronic heat capacity and $C_p = \beta T^3$ is the heat capacity from the lattice (phonons) with both γ and β material dependent constants. The parameter β (J K⁻⁴ m⁻³) is found from Debye theory and can be expressed as:

$$\beta = 1.944 \times 10^6 \, \frac{\rho}{M} \frac{1}{\Theta^3} = \frac{A}{\Theta^3} \,, \tag{6.3}$$

where ρ is the density (kg m⁻³) of the absorber, *M* is the molecular weight (g Mole⁻¹) and Θ is the Debye temperature. The parameter γ (J K⁻² m⁻³) is found from free electron theory [Hook and Hall, 1999]:

$$\gamma = \frac{\pi^2 n_e k_b}{2T_F} \,, \tag{6.4}$$

where T_F is the Fermi temperature, n_e is the number of free electrons per unit volume and k_b is Boltzmann's constant.

Metallic absorbers are most commonly used for TES detectors. It is, however, also possible to use superconducting and dielectric absorbers, as in early semiconductor thermistor development. Each of these absorber types is reviewed below.

6.1.1. Metal and Semimetal Absorbers

Metal absorbers are ideal for TES detectors from the point of view of high detector efficiency and rapid thermalisation; however they are also characterised by a high heat capacity. For a metal at mK temperatures the lattice contribution will generally be negligible and the first term of Equation 6.2 will dominate. Gold, copper and semi-metals such as bismuth or antimony are commonly used materials for TES X-ray absorbers. In the case of Bi, the low electron carrier density of ~ 10^{17} cm⁻³ [Williams, 1965] (as opposed to ~ 10^{22} cm⁻³ for a normal metal), means that the heat capacity contribution from the electronic component is much smaller than for a metal and is in fact comparable to the lattice component at mK temperatures [Collan et al., 1970].

Property	Units	Au	Cu	Bi
Atomic Number, Z	-	79	29	83
Molecular Weight, M	g Mole ⁻¹	196.665	63.546	208.91
Density, ρ	kg m ⁻³	19300	8920	9800
Sommerfield constant, γ	J K ⁻² m ⁻³	65.74	97.53	3.93
Debye Temperature, Θ	K	165	343	119
A/Θ^3	J K ⁻⁴ m ⁻³	42.47	6.76	54.11
Electronic specific heat at 120 mK, C_e	J K ⁻¹ m ⁻³	7.89	11.70	0.47
Lattice specific heat at 120 mK, C_p	mJ K ⁻¹ m ⁻³	73.4	11.7	93.5
Total specific heat at 120 mK, C	J K ⁻¹ m ⁻³	7.96	11.72	0.57
Mass absorption coefficient at $E = 5.9$ keV, α	$m^2 kg^{-1}$	44.3	12.1	50.6
Thickness required for 90% QE at $E = 5.9$ keV, d	μm	2.71	21.3	4.64
Absorber heat capacity $(250 \times 250 \times d \ \mu m^3)$, C_{abs}	рЈ К ⁻¹	1.34	15.6	0.14
Energy resolution ratio to 1 pJ K ⁻¹ TES	-	1.54	4.06	1.08

Table 6.1. Summary of properties for Au, Cu and Bi.

Table 6.1 summarises the properties of these elements and also lists the total thickness and heat capacity required to achieve 90 % QE at a photon energy of E = 5.9 keV. The heat capacity is estimated using Equation 6.2. The values of γ are experimental estimates tabulated by Ashcroft and Mermin (1976). The Debye temperatures are taken from Kittel (1996) and the constant A derived using Equation 6.3. The heat capacity of the absorber assumes typical microcalorimeter dimensions of $250 \times 250 \times d \,\mu\text{m}^3$. The energy resolution ratio is calculated assuming a typical TES with $C = 1 \,\text{pJ K}^{-1}$ plus the absorber heat capacity C_{abs} , divided by the energy resolution assuming just the heat capacity of the TES (i.e. no absorber). Thus we see that in terms of intrinsic resolution and QE, the best
performance is achieved using a bismuth absorber; 90 % QE is possible with only 1.08 times the resolution of a TES with no absorber. However, the disadvantage of using bismuth lies in its low thermal conductivity at cryogenic temperatures. The thermal conductivity of a metal κ , can be estimated from kinetic theory [Hook and Hall, 1999]:

$$\kappa = \frac{1}{3} C_e v_F \lambda_e \,, \tag{6.5}$$

where v_F is the Fermi velocity (defined as the velocity of electron in states on the Fermi surface and is typically ~ 10⁶ ms⁻¹), λ_e is the electron mean free path and C_e is electronic heat capacity. In thin films at low temperature the electron mean free path will be limited by impurity scattering and boundary scattering, consequently it is reasonable to assume that λ_e will be limited by the smallest dimension of the absorber, if the film purity is high enough and the surface scattering is diffuse. In this régime, the only temperature dependence comes from the heat capacity (Equation 6.2), so that κ scales directly as *T*. For Bi, the low electron density, which gives rise to the low heat capacity (Equation 6.4), also results in a low thermal conductivity and hence the diffusion time of the absorber could be greater than the TES effective time constant. Thus spatial dependence of the photons in the absorber is a potential consequence resulting in the degradation of the energy resolution. Similarly for a DROID detector we have seen in Chapter 4 that the thermal conductance of the absorber affects both the energy and position resolution, with rapid thermalisation important for best energy resolution performance. For TES detectors, a Bi absorber is often used to provide the required stopping power, while a separate underlying heat diffusion layer of Cu or Au is used to provide rapid thermal diffusion to the TES [Iyomoto et al., 2004].

6.1.2. Superconducting Absorbers

Superconducting materials are also possible candidates for microcalorimeter absorbers. The heat capacity of a superconductor differs from that of a normal metal. At the superconducting to normal transition, where the energy gap Δ appears, there is a jump in specific heat associated with the second order phase change [Bardeen et al., 1957]. As with a normal metal, the superconductor heat capacity consists of an electronic contribution and a lattice/phonon contribution. However differences in the specific heat of the normal and superconducting states are attributed to differences in the electronic contribution made that the phonon contribution C_p (of Equation 6.2) remains the same for both states [Corak et al., 1956]. It is found that the electronic contribution contains an exponential term [Bardeen et al., 1957],

$$C_{es} = \gamma T_C a e^{\frac{-b}{t}}, \qquad 6.6$$

in which *a* and *b* are constants, and $t = T/T_c$ is the reduced temperature. The constants *a* and *b* are themselves dependent upon the reduced temperature *t*. The parameter *b*, is a function of the temperature dependent energy gap of the superconductor $\Delta(T)$,

$$b = \frac{\Delta(T)}{2k_b T_C} \,. \tag{6.7}$$

At t = 0 and at t = 1 the Bardeen-Cooper-Schrieffer (BCS) model predicts that the constants *a* and *b* will be the same for all superconductors [Phillips, 1959]. The universal result of the BCS theory is that at the transition (t = 1), there will exist a discontinuity in the electronic heat capacity, such that just below the transition edge the electronic heat capacity will be 2.43 times greater than in the normal state [Ashcroft and Mermin, 1976]:

$$C_{es} = 2.43\gamma T_C$$

= 2.43C...
6.8

However for $T \ll T_C$ the exponential drop off in the electronic contribution means that the lattice contribution will dominate, especially for low Θ superconductors (such as Hg for example, where $\Theta = 71.9$ K). The total heat capacity is therefore reduced below the normal state value and the specific heat will scale as T^3 , as for a dielectric. This is advantageous from the point of view of designing an absorber where we want to minimise the overall heat capacity for optimised resolution.

When a photon is absorbed by a superconductor the energy deposited produces a phonon distribution. Phonons with energy Ω greater than the superconducting energy gap ($\Omega \ge 2\Delta \sim meV$) are able to break up Cooper pairs into electronic excitations known as quasi-particles. This process is analogous to the electron-hole pair dissociation in semiconductors, though because the band gaps are smaller in the superconducting case; many more quasi-particles are created for a given energy. Excited quasi-particles recombine by emission of another phonon, which in turn can create further quasi-particles [Rothwarf and Taylor, 1967] or escape to the TES. Lower energy phonons which do not contribute to pair dissociation relax to a thermal distribution via scattering processes with other quasi-particles [Chang and Scalapino, 1977]. Phonons detected by the TES contribute to the output signal. Thermal equilibrium is reached when the pair-dissociation rate is equal to the rate of recombination. Statistical fluctuations in quasi-particle recombination between each X-ray event will affect the output signal and act to degrade the detector energy resolution. A further problem with the use of superconducting absorbers is that at temperatures very much below the transition temperature, the recombination time and hence the thermalisation time, can be very slow (> ms) [Booth and Goldie, 1996; Stahle et al., 1994]. This is because at low temperatures the quasi-particle background population available for

recombination is low, consequently, many of the phonons produced go into creating further quasiparticles rather than being detected by the TES.

Direct absorption in the TES itself is also possible, however the thin film thickness ($d \sim 100$ nm) required as part of the design constraints mean that the QE will inevitably be small. The physics of the transition state is complex and it is possible that the TES may contain domains with differing phase states. Evidence from direct absorption in Ir TESs [Ohkubo et al., 2002] and Ir/Au bilayer TESs [Kunieda et al., 2004] shows a spatially varying response to X-ray events. For the Ir TES, two-dimensional absorption features are associated with a non-uniform supercurrent distribution due to self-heated resistive domain formation. Spikes observed on the leading edge of the standard pulse are found to be more pronounced in regions of the TES where the supercurrent is largest and are associated with hot spot formation. For the Ir/Au TES the thermalisation is improved by the addition of the Au layer; however spatial variations in pulse shape are still found. Such spatial variations are undesirable and degrade the energy resolution of the device, however they do open up the possibility of developing a position sensitive TES [Ohno et al., 2004].

6.1.3. Dielectric Absorbers

Dielectric absorbers, in particular crystalline silicon, have also been investigated for use as microcalorimeter absorbers [Leman et al., 2004; Young et al., 1990], for both X-ray and particle detection, because of their intrinsically low heat capacity. The initial photoelectron produced after an X-ray interaction in silicon will break apart many electron-hole pairs. Similarly, charged particles (α -particles and protons) created by nuclear interactions in fast (MeV) neutron spectroscopy (for example), will deposit almost 100 % of their energy via ionisation along the particle track. The electrons and holes created in these processes decay by phonon emission and usually recombine at the crystal surface, where there are numerous deformations and impurities. The emitted phonons that propagate into the detector then make up the measured signal. As with superconducting absorbers, a 100 % recombination rate may not occur in crystalline counterparts. Some of the deposited energy may become trapped in impurities or long lifetime electron-hole pairs. Such metastable states thus result in incomplete absorber thermalisation over short time scales and degrade the detector resolution [McCammon et al., 1986]. Frank et al. (1994) present evidence that such effects do not adversely affect detector resolution in a high purity silicon absorber used for particle detection, though similar results obtained using sapphire of a low purity apparently do show this effect [Seidel et al., 1990].

In a dielectric crystal the heat capacity is dominated by the lattice contribution and thus the second term in Equation 6.2 dominates. For silicon this reduces to:

$$C = 0.59T^3 \,\mathrm{J} \,\mathrm{K}^{-1} \,\mathrm{m}^{-3}.$$

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This T^3 relationship for a dielectric means that at temperatures below 1 K the heat capacity drops off much more rapidly than for a metal, where the heat capacity scales with *T*. For 90 % QE at E = 5.9keV, a thickness of $d = 64 \,\mu\text{m}$ of Si is required, which will only contribute ~ f J K⁻¹ at T = 0.1 K to the overall heat capacity. This is ideal for high energy resolution spectroscopy, though for the detection of α -particles or neutrons where the incident energy maybe several MeV, absorbers with C > nJ K⁻¹ are required. Such heat capacities will dominate over the TES specific heat, consequently large volumes may be required to ensure that temperature excursions are not so large as to saturate the device.

The propagation of energy from a dielectric absorber to the TES is dominated by the phonon thermal conductivity of the absorber κ (W m⁻¹ K⁻¹), which as for a metal, can be estimated from kinetic theory [Hook and Hall, 1999]:

$$\kappa = \frac{1}{3} C_p \nu \lambda_p , \qquad 6.10$$

where C_p is the heat capacity, ν is the average speed of sound over all phonon modes, which is approximately equal to the Debye speed of sound ($\nu = 5660 \text{ m s}^{-1}$ in Si [McCurdy, 1970]), and λ_p is the average phonon mean free path. In any solid, phonons will decay by anharmonic processes (phonon-phonon collisions), defect and impurity scattering, as well as by isotope scattering [Maris, 1990]. There are two types of three-phonon anharmonic scattering, Normal (N) and Umklapp (U) [Kittel, 1996] – only U scattering is responsible for providing thermal resistance, whilst the N scattering causes frequency conversion and mode conversion (polarisation) of the phonons. At low temperatures the population of high-energy phonons able to participate in U scattering rapidly decays, hence phonon-phonon collisions become ineffective in providing thermal resistance. In this régime λ_p becomes very large (~ cm) and can exceed the specimen size. The phonon mean free path is therefore limited by the dimensions of the absorber and the quality of the crystal surfaces. If the crystal is of high purity and surface scattering is specular, then ballistic phonon transport of low energy phonons can occur. Diffuse scattering of phonons by impurities and surface roughness can reduce the thermal conductivity below the specular limit [Casimir, 1938]. If diffuse surface scattering occurs then $\lambda_p = d$.

Energy deposition in a Si absorber will create an excited distribution of phonons with an average energy corresponding, approximately, to the Debye energy (~ 60 meV for Si) with a characteristic Debye frequency of $v_D \sim 13$ THz. These phonons will be subject to spontaneous decay via anharmonic processes (LA \Leftrightarrow TA + TA and LA \Leftrightarrow LA + TA, where LA are longitude acoustic modes and TA are transverse acoustic modes) and have life times of 10-100 ps [Maris et al., 1993] thus their mean free path is very small (100 nm) compared to the specimen size. The average lifetime of the phonons becomes longer as the phonons down convert since the anharmonic decay rate τ_A^{-1} rapidly slows with decreasing frequency ν . Averaged over all modes this can be expressed as [Maris, 1990],

 $\tau_A^{-1} = Av^5$, where $A = 1.2 \times 10^{-55} \text{ s}^3$ [Esipov, 1994]). Si does not exist as a single isotope thus phonons will also scatter off isotopic impurities at a rate τ_l^{-1} (in the low frequency limit $\nu \ll \nu_D$) given as $\tau_I^{-1} = B \nu^4$ [Maris, 1990], where $B = 2.43 \times 10^{-42} \text{ s}^3$ [Esipov, 1994]. Assuming that the surface reflection is specular, ballistic transport will occur as λ_A becomes larger than the crystal dimensions. Quasi-diffuse [Esipov, 1994; Tamura 1993; Maris et al., 1993; Bron et al., 1982; Wilson et al., 1984] transport can arise, in which both ballistic and diffuse components exist, if the elastic isotopic impurity scattering rate is comparable to the anharmonic scattering rate $\tau_l^{-1} \sim \tau_A^{-1}$ (with both process occurring several times over an experimental time scale). This process has been extensively investigated in Si with ballistic contributions experimentally observed in α -particle, X-ray and neutron experiments [Cabrera et al., 1993]. For example, observations of ballistic phonon transport in a Si absorber with a Ti TES [Young et al., 1990; Young et al., 1992] due to α -particle interactions, show ~ 1/3 of the transported phonons in a 1 mm crystal were found to traverse the crystal ballistically. Phonon propagation is further complicated by a dependence upon the crystallographic orientation of the specimen. The ballistic energy flow can be enhanced in some directions and suppressed in others by phonon focussing [Taylor et al., 1969] resulting from elastic anisotropy of the crystal lattice. The thermal conductance consequently depends on the orientation of the absorber axis [McCurdy et al., 1970]. For Si it is shown that transverse phonons are most strongly focussed in the [100] direction. The work by Peterreins et al. (1988), using superconducting tunnel diodes on a Si wafer absorber, demonstrate the possibility of using phonon focussing patterns to obtain spatial resolution.

6.2. Prototype DROID Design

Our DROIDs, designed for soft X-ray detection, consists of two Ir TESs at either end of a continuous, linear Au or composite Au/Bi absorber. In the latter composite absorber, the semi-metal Bi is used to provide the stopping power for the photons and an underlying Au layer is used to provide effective thermal diffusion to the TESs. We currently have two different prototype DROID configurations fabricated using magnetron sputtering by Oxford Instruments Superconductivity (OIS) under contract to the University of Leicester in 1998. The full details of the processing steps used to fabricate the devices are described by Trowell (2001). These devices are formed on a single $15.1 \times 15.1 \text{ mm}^2$ Si die, $375 \mu \text{m}$ thick. The Si die is manually mounted in a Cu holder using Ag loaded epoxy (EPO-TEK H20E) as the adhesive, on four small tabs, one on each edge of the die (see Figure 6.1). This mounting scheme was specially designed to allow for differences in thermal contraction of the Cu mount and the Si die, which in previous design schemes had resulted in the Si die breaking under stress.



Figure 6.1. Photograph of the reverse side of a Si die mounted in a $30 \times 30 \ \mu m^2$ gold plated copper holder. Clearly visible are the four tabs attached to the die via the silver loaded epoxy [Trowell, 2001].

Two 500 nm thick Si_3N_4 membranes provide the weak thermal link between the DROIDs and the heat bath. The first device on the die consists of a 4.32×0.25 mm² absorber with both TESs constructed fully on the membrane. The second device has its TESs half on the membrane and half thermally anchored on the thick substrate; the absorber length is then 4.75 mm. This second configuration aimed to improve the thermal conductivity from the TES to the cold bath and hence the maximum count rate of the device. The Ir TESs, at both ends of the absorber, measure $250 \times 250 \ \mu\text{m}^2$ and are 100 nm thick.

Figure 6.2 shows an optical micrograph of a prototype DROID die showing the two different configurations described above. This device consists of just the Au diffusion layer (210 nm) without the full Bi absorber. Shown on the absorber are two thick Au (880 nm) blocks which mean to provide position calibration, by means of differential absorption at known x/L values. Also shown are resistor meanders for measuring the residual resistivity ratio (RRR) and structures for measuring intermetallic/ageing effects [Trowell, 2001]. Figure 6.3 shows a close up optical micrograph of one end of a DROID with Au/Bi absorber, here some of the Bi has spilled onto the membrane. A single pixel Ir/Au proximity bilayer TES accompanies each DROID on the membrane. A bilayer TES consists of two different overlaying thin films with different critical temperatures. Because of the proximity effect [Martinis et al., 2000], depending upon the relative thicknesses of each layer, the actual transition temperature of the bilayer will be somewhere in between the transition temperature of the two materials.



Figure 6.2. Optical micrograph of two prototype DROID configurations on a single detector die. These particular prototypes have two thick blocks at known positions on the absorber to provide position calibration by means of enhanced absorption.

The TESs are designed to be biased using gold (880 nm) on-chip shunt resistors of resistance $R_s = 35$ m Ω . The bias leads between the TESs and shunt resistors is provided by thin (225 nm) superconducting niobium ($T_c = 9.3$ K) tracks. Due to a design error on the original mask one bias line was missing at either end of the DROID absorber, consequently a superconducting Al ($T_c = 1.2$ K) wire bond (Figure 6.2) had to be attached to enable correct biasing of the TESs. The bias rails for the DROID TESs are designed such that the current return path is on the absorber-TES interface (Figure 6.3). Extending the bias rail over the complete absorber-TES interface would lead to a thin superconducting interface (effectively a Nb/Ir bilayer) between the TES and absorber, which would be thermally non-conductive [Lynton, 1969] and impede heat transfer from the absorber to the TES. However, in making the electrodes different sizes, we introduce a non-uniform current density in the

TES which could cause effects such as phase separation possibly leading to 'excess' TES noise [Luukanen A. et al., 2003] and would be detrimental to performance. Future designs will see the TES contact electrodes running the full length of the TES sides, parallel to the absorber axis, so that the current path is perpendicular to the absorber axis.



Figure 6.3. Optical micrograph of a prototype DROID with composite Bi/Au absorber. At one point of the absorber the Bi has spilled on to the membrane. This only occurred on one of the two Bi/Au DROIDs.

6.3. Thin Film Material Properties

The dominant scattering of conduction electrons in a pure metal is by phonons and is highly temperature dependent with a scattering rate $\tau(T)^{-1}$, tending to infinity as the temperature reduces to zero. In real metals electrons will also scatter off impurities, structural defects and grain boundaries at a rate τ_0^{-1} , which is independent of temperature. Using Mattheisen's rule, independent scattering mechanisms can be combined [Hook and Hall, 1999] so that the total electron scattering rate τ^{-1} is expressed as the sum of the temperature dependent scattering rate $\tau(T)^{-1}$ and the zero-temperature scattering rate τ_0^{-1} due to impurities and due to defects, $\tau^{-1} = \tau(T)^{-1} + \tau_0^{-1}$. These scattering rates are inversely proportional to the electrical resistivity, so the total electrical resistivity ρ is, $\rho = \rho(T) + \rho_0$, where $\rho(T)$ is now the temperature dependent resistivity and ρ_0 is residual resistivity. As $T \to 0$, electron-phonon collisions become frozen out and measurements of the resistivity at 4.2 K, $\rho_{4.2}$, give a good estimate of the residual resistivity ρ_0 . The residual resistivity ratio (RRR) is then defined as the ratio of the resistivity of the material measured at room temperature (~ 300 K), to that measured at 4.2 K.

$$RRR = \frac{\rho_{300}}{\rho_{4.2}} \,. \tag{6.11}$$

 $\rho_{4.2}$ and the RRR are important design parameters which give information on the purity and therefore the electrical and thermal characteristics of the film at cryogenic temperatures. In sufficiently high purity thin films, the mean free path of impurity scattering may actually be longer than the thickness of the film and boundary scattering will dominate (the mean free path then depends on the quality of the material surfaces). Table 6.2 shows the resistivity measurements taken using the four on-chip DROID RRR resistors shown in Figure 6.2.

Table 6.2. Resistivity measurements for DROID meanders measured at 300 K and 4.2 K using a fourpoint AVS-47 resistance bridge. The Niobium resistor was measured at ~ 9.3 K before going superconducting. Also shown are the calculated electron mean free paths λ_e for the two Au layers.

Structure	Material	<i>d</i> (± 5 nm)	ρ_{300} ($\mu\Omega$ cm)	$\rho_{4.2}$ ($\mu\Omega$ cm)	RRR (± 0.01)	λ_{e} (nm)
Tracking	Nb	225	20.22 ± 0.45	4.76 ± 0.11	4.25	-
TES	Ir	100	13.24 ± 0.66	7.75 ± 0.39	1.71	-
Absorber	Au	210	4.42 ± 0.11	1.94 ± 0.05	2.27	44.5 ± 1.1
Shunt	Au	880	4.24 ± 0.02	1.67 ± 0.01	2.54	51.7 ± 0.3

6.3.1. Absorber Properties

The dependence of the Au film thickness d on $\rho_{4,2}$ is an indicator that boundary scattering processes may be important. The electrical resistivity of the film can be expressed as [Hook and Hall, 1999]:

$$\rho_0 = \frac{m_e v_F}{n_e e^2 \lambda_e} \,, \tag{6.12}$$

where λ_e is the electron mean free path, m_e is the mass of an electron, n_e is the electron density, e is the electron charge and v_F is the Fermi velocity. Using tabulated data from Ashcroft and Mermin (1976) and our measured values for ρ_0 it is possible to calculate the mean free path λ_e ; as shown in Table 6.2. For these Au samples $\lambda_e < d$, but still with some dependence upon d, thus impurity scattering and grain boundary scattering in these films is likely to be the more dominant process over surface scattering.

For a DROID absorber we have seen from the modelling in Chapter 4 that a low heat capacity coupled with a high thermal conductance is required for optimum energy resolution. The thermal conductance of the absorber is directly related to the electrical resistivity via Equations 6.5 and 6.12, consequently a low electrical resistivity is required for high thermal conductivity. 4.2 K resistivity measurements made at the Cavendish Laboratories [Goldie, 2004] on thin film Cu of a variety of thicknesses, demonstrates an order of magnitude reduction in resistivity over the Au films used in the prototype DROIDs. These findings are summarised in Table 6.3. Estimating the electron mean free path λ_e , we find that in these samples, $\lambda_e \sim d$ and is in fact slightly longer than d, illustrating that boundary

scattering (predominately diffuse) dominates over the impurity scattering and grain boundary. Consequently larger RRR values are achieved than in the Au films.

Table 6.3. Resistivity measurements for copper resistors deposited by magnetron sputtering [Goldie, 2004]. Also shown is the calculated electron mean free path λ_{e} .

<i>d</i> (nm)	ρ_{300} ($\mu\Omega$ cm)	$\rho_{4.2}$ ($\mu\Omega$ cm)	RRR	λ_{e} (nm)
30	3.74	1.86	2.01	36
50	2.64	0.86	3.08	78
206	1.46	0.21	9.66	321

In Table 6.4, we calculate, from Equations 6.2 and 6.5 using the data in Tables 6.1, 6.2 and 6.3, the thermal conductance G_{abs} and the heat capacity C_{abs} , of a 4.32 mm DROID absorber. The implication is that the same absorber heat capacity as used in the current DROIDs can be achieved with a much greater thermal conductance. Alternately the same thermal conductance can be achieved with a much reduced heat capacity. In Table 6.4 we also calculate, using the finite-element model and algorithms developed in Chapter 4, the FWHM energy resolution at the centre of the DROID, all other parameters being kept the same as for DROID 1 (listed in Table 3.1). The data in Table 6.4 shows that at all thicknesses investigated, a Cu absorber will produce better energy resolution performance than the Au equivalent used in the current prototype design. Consequently, in future DROID designs we aim to use Cu as the heat diffusion layer.

Table 6.4. Thermal conductances G_{abs} and heat capacities C_{abs} for a 4.32 mm DROID 1 absorber using different materials and thicknesses. Also included is the important design parameter $g = G_{abs}/G$ (see Chapter 4) and the calculated energy resolution at the centre of the DROID absorber.

Material	<i>d</i> (nm)	<i>G_{abs}</i> (nW K ⁻¹)	$g = G_{abs}/G$	C _{abs} (pJ K ⁻¹)	$\Delta E(x=L/2)$ (eV)
Au	210	2.3	0.84	2.6	18.4
Au	880	13.2	4.82	11.1	23.7
Cu	30	0.4	0.15	0.6	14.9
Cu	50	1.5	0.55	0.9	12.7
Cu	206	24.6	8.98	3.8	12.4

6.3.2. Iridium and Iridium/Gold TESs

A low bulk transition temperature $T_C \sim 112$ mK, high corrosion resistance and good long term chemical stability make iridium a promising candidate for use as TES detectors. Ir has been investigated by numerous groups [Pergolesi et al., 2004; Fukuda et al., 2000, 2002; Von Kienlin et al., 1998; Frank et al., 1994] involved in TES detector development and is the material used in our DROID TESs. Resistance versus temperature (*R*-*T*) measurements at the University of Leicester on thin film Ir resistors, deposited directly on a silicon nitride substrate, reveal a tuneable T_c , which is dependent upon the substrate deposition temperature [Trowell, 2001]. The transition temperatures for the six samples (deposited under different conditions) lie in the range 135 to 157 mK, except for one sample for which $T_C = 52$ mK. Fukuda et al. (2000) report $T_C = 326$ mK for an Ir film sputtered onto an Si substrate and $T_C = 289$ mK of Ir sputtered on Si₃N₄. The generally higher T_C than for bulk Ir is thought to be a result of inter-diffusion between the Ir and Si layers, resulting in an iridium silicide sub-layer. This process is not well understood but thought to be consequence of heating the substrate as part of the deposition process [Trowell, 2001; Fukuda et al., 2000; Petersson et al., 1979]. Ir RRR measurements using the on-chip DROID resistors give values consistent with measurements by Fukuda et al. (2000) for Ir on Si_3N_4 (RRR = 1.71) and the Ir resistors tested by Trowell (2001) (RRR = 1.71-1.74). We have also re-tested a small subset of these Ir meander resistors and found no change in the resistivity values over a two-year period. This is an indicator that inter diffusion processes are not likely to have occurred on these samples (stored at room temperature) over this period. Measurements of T_C and the shape of the *R*-*T* transition do, however, reveal changes which warrant further testing. It is not possible to completely eliminate experimental differences in the testing of the samples between the two runs. For example, magnetic shielding was not fitted on the latter run but was for the initial testing. Thus, it is possible that differences in stray magnetic fields may have affected the shape and temperature of the transition [Takei et al., 2004]. We will be re-testing these samples with magnetic shielding present in the near future.

TES ID	$R_n (\pm 1 \text{ m}\Omega)$	Layer ($d (\pm 5 \text{ nm})$)	Measured $T_C (\pm 5 \text{ mK})$
112/2 DROID 1 TES 1	753	Ir (100)	-
112/2 DROID 1 TES 2	763	Ir (100)	176 mK
112/2 DROID 2 TES 1	750	Ir (100)	177 mK
112/2 DROID 2 TES 2	758	Ir (100)	177 mK
112/2 Bilayer TES 1	17	Ir (100)/Au (880)	< 60 mK
112/2 Bilayer TES 2	17	Ir (100)/Au (880)	< 60 mK
113/1 DROID 1 TES 1	893	Ir (100)	-
113/1 DROID 1 TES 2	936	Ir (100)	-
113/1 Bilayer TES 1	98	Ir (100)/Au (210)	76 mK

Table 6.5. Normal state resistances R_n and transition temperature T_c , for Ir and Ir/Au TESs measured at the University of Leicester in the top-loading DR using an AVS-47 4-point resistance bridge.

Two different detector dies have been investigated with normal state resistances R_n and Transition temperature T_c as listed in Table 6.5. The Ir TESs generally have a higher resistance (consistent with the resistivity meander measurements in Table 6.2) compared to TESs being developed by other groups, which typically have $R_n \sim 100$'s m Ω . A high TES resistance means that the thermal conductivity of the TES will be low. A low TES thermal conductance increases the likelihood of resistive hotspot formation resulting in phase separation of the TES. This is even more a concern in these TESs due to the likely non-uniform current density in the TES (Section 6.2). Irwin et al. (1998) show that the effects of self heating on a TES should not be a problem so long as the following condition is met:

$$R_n < \frac{L_n T_C}{G} \left(\frac{n}{\alpha}\right),\tag{6.13}$$

where $L_n = 24.5$ nW Ω K⁻² is the Lorentz number and is a constant [Kittel, 1996]; all other terms have their are defined in Table 3.1. Using the data from Table 3.1, we require $R_n < 0.16 \Omega$ for a DROID 1 configuration TES and $R_n < 0.11 \Omega$ for a DROID 2 configuration TES, confirming that for these high resistance TESs, self heating and phase separation are probable. Phase separation is unwanted and can result in excess noise [Luukanen et al., 2003] and position dependence of photons in the absorber. Pressler et al. (2002) demonstrate, using low temperature scanning synchrotron microscopy (LTSSM), that phase separation in their 500 μ m × 500 μ m × 50 nm thick Ir TES occurs due to a small TES thermal conductance and high normal state resistance. The current signals due to X-ray absorption are spatially dependent indicating that parts of the TES are resistive while others remain superconducting.

The measured T_C for the Ir TESs was higher than that for the original resistor samples tested by Trowell (2001) but within the spread of experimental data by other authors. Ir/Au proximity bilayers have been invested by numerous groups [Nagel et al., 1994; Stark et al., 2004, Kunieda et al., 2004], with T_C varying from < 30 mK to 110 mK. The added Au layer has the advantage of increasing the thermal conductance of the TES, to limit the effect of spatial variation in the absorber as well as enabling a tuneable T_C . Kunieda et al. (2004) show that thermalistation is improved by adding a 25 nm layer of Au to their 100 nm TES ($T_C = 110$ mK), but LTSSM measurements reveal spatial dependence in the absorber still exists. The T_C for our Ir(100 nm)/Au(210 nm) TES was measured to be approximately 76 mK. On the particular run to measure the transition temperature of the Ir(100 nm)/Au(880 nm) TES, the dilution refrigerator base temperature could not be reduced below 60 mK and no transition was observed.

Another effect of the additional Au layer on the TES is to reduce the normal state resistance of the device by effectively shunting the Ir layer. Using the resistivity measurements in Table 6.2 we can predict R_n for the Ir/Au TESs, which gives $R_n = 18.5 \text{ m}\Omega$ for Ir(100 nm)/Au(880 nm) and $R_n = 82.5 \text{ m}\Omega$ for Ir(100 nm)/Au(210 nm), consistent with the experimental measurements. Note that it is not possible to achieve correct voltage bias of these detectors using the 35 m Ω on-chip shunt resistors. Applying Equation 6.13 to the Ir (100 nm)/Au (210 nm) TES, assuming estimated values of n = 4.1, $G = 2.8 \text{ nW K}^{-1}$, we require $R_n < 90 \text{ m}\Omega$ for self heating effects not to be a problem. Thus, phase separation is again a possibility but much less of a problem compared with the single layer Ir TESs.

6.4. Thermal Characterisation of the DROID

The overall thermal conductance of the weak link from the TES to the cold bath is an important parameter that affects device count rate and in the case of a DROID, can also affect both energy and position resolution (see Chapter 4). The DROID design consists of multiple thermal links from the TES to the mixing chamber of the DR, which need to be characterised.

The top loading ³He/⁴He dilution refrigerator (see Section 5.3.3) was used to cool the device to below its superconducting transition temperature of ~ 177 mK. The first device investigated was device 112/2 DROID 2 TES 1 (Table 6.5), deposited half on the membrane and half on the substrate (Figure 6.1). Note that due to the availability of only a single SQUID at the time of testing, the second TES at the other end of the DROID absorber was not wired up. Hence the measured thermal characteristics will be different from those when the DROID is used in dual channel operation. The device was DC biased at the top of the transition edge, just in the normal state. Thus, knowing the shunt resistor (35 m Ω), the normal state resistance of the TES (0.75 Ω) and the series resistance (6786 Ω), the power dissipated in the TES could be calculated. By varying the DR base temperature T_b , the power required to hold the same point on the transition edge changes. Measuring the power as a function of T_b (measured by the mixing chamber RuOx calibrated thermometer), and fitting to the data using the power law Equation 2.9, we can determine the thermal conductance to the bath *G*, as well as the constants *K* and *n*. Measurement of these parameters indicates the physical processes which control the heat flow from the TES to the cold bath. *G*, *K* and *n* values are listed in Table 6.6 with the fit to the data shown in Figure 6.4.



Figure 6.4. Power plateau measurements for a DROID TES half on the membrane and half on the substrate. The TES at the other end of the absorber was not connected.

Table 6.6. Thermal conductivity measurements of a DROID TES half on the membrane and half on the substrate.

Parameter	112/2 DROID 2 TES 1
T_C	$176.9 \pm 0.3 \text{ mK}$
K	$114.5 \pm 0.1 \text{ nW K}^{-n}$
п	3.40
G	$6.12 \pm 0.03 \text{ nW K}^{-1}$

From Figure 6.4, it is apparent there is little variation in the power require to bias the TES for bath temperatures of ≤ 60 mK. By inspection of Equations 2.15 and 2.16 the implication, therefore, is that over this temperature range the loop gain L_0 and therefore the electrothermal feedback, decay time constant and the energy resolution will also all be un-affected. Thus, as long as the cooler remains at a base temperature below 60 mK, the detector performance should not be adversely affected.

One possible flaw in the current DROID design lies in the close proximity of the TESs and the shunt resistors (~ 750 μ m). The shunt resistors typically dissipate an order of magnitude more power (nW's depending upon the bias point) than the TESs (several 100 pW's) themselves. This may cause localised heating in the TES substrate, leading to an effective base temperature which is in fact higher than that measured on the mixing chamber thermometer. This would mean that the estimated *G* would be less than the actual *G*, since the power input required to self heat the TES would be smaller due to the additional heating effect.

On a later run we re-tested the DROID TES using off-chip biasing to remove any possible heating effects from the original shunt. Due to SQUID failure before this run, the detector current and voltage had to be measured using in-house amplifiers (developed by Mr. C. Whitford). The voltage was measured directly across the TES and the current was calculated by measuring the voltage across a small resistor of known value in series with the TES. The results are noisier and give slightly different values for the fit parameters than the data previously obtained using SQUID read-out. As opposed to the SQUID testing, the power measurements here were made at the at the bottom edge of the transition (since this was more clearly defined), which we assume to be at $R_0 \sim R_l = 35 \text{ m}\Omega$ where the voltage bias is lost. The Joule power is ~ 20 % lower at the bottom of the transition than at the top, which in turn we estimate will result in an estimated *G* value no more than ~ 20 % lower at the bottom as well. In this run we also tested a TES on DROID 1 which was mounted completely on the membrane. The parameters for both the DROID 1 and DROID 2 configurations are listed in Table 6.7.

Note that the calculated values for *G* and *K* for DROID 2 TES 1 are slightly less than those listed in Table 6.6, possibly because the measurements of *G* were evaluated at different points in the transition. These results do not suggest that any localised heating effects are adversely affecting the conductivity measurements. There is a factor of \sim 1.6 difference in *G* values between the two TES configurations,

confirming that the increased coupling of the TES half mounted on the substrate does indeed increase the thermal conductance to the bath.

Table 6.7. Fitted thermal parameters for DROID TESs in the two different configurations measured using the in-house built amplifier chain. Also shown is the data for the single pixel Ir(100)/Au(210) TES measured at the Cavendish Laboratory using SQUID read-out.

Parameter	112/2 DROID 1 TES 2	112/2 DROID 2 TES 1	113/1 Bilayer TES 1
T_C	174.7 ± 9.6 mK	175.8 ± 4.8 mK	$75.7 \pm 8.2 \text{ mK}$
K	$17.9 \pm 2.5 \text{ nW K}^{-n}$	$68.7 \pm 6.1 \text{ nW K}^{-n}$	$2.1\pm0.9~\mu W~K^{\text{-n}}$
п	2.63 ± 0.09	3.27 ± 0.05	4.1 ± 0.2
$G(T_C)$	$2.74 \pm 0.46 \text{ nW K}^{-1}$	$4.33 \pm 0.47 \text{ nW K}^{-1}$	$2.8 \pm 1.6 \text{ nW K}^{-1}$

Devices provided by SRON (Space Research Organisation Netherlands) (Dr. Henk Hoevers), for testing the overall thermal conductivity of the Leicester mounting scheme, consisted of a Ti/Au TES and a heater in close proximity on a Si substrate. As with the DROID devices, the Si was attached (by Mr. R. Limpenny) to the Cu mount using a small amount of epoxy on each of the four tabs. By measuring the heater power, as a function of base temperature required to drive the TES into the normal state, it is possible to estimate the thermal conductivity of the structure. Two identical devices were tested in different mounts; the heater power measurements as a function of base temperature are plotted in Figure 6.5 for both samples with the fit parameters listed in Table 6.8. Also shown in Table 6.8 is the equivalent G at T = 177 mK for a direct comparison with the Ir TES data. The differences in the measured thermal conductance G are attributed to differences in the thickness of the epoxy layer used to attach the Si to the Cu mount. These conductance values are much larger than the measured conductance for the TES to cold bath, indicating that the mounting structure is not the limiting conductance in the system. A 32×8 mm DROID array would give focal plane coverage equivalent to a NFI for XEUS. Each DROID consists of two TESs each dissipating ~ 300 pW of power implying a total power dissipation of ~ 20 nW, whereas using the data in Table 6.8 the mount could dissipate between 25 and 50 nW at 177 mK. This configuration may be just adequate for DROID array development, though with limited margin for error. This illustrates the advantage in thermal engineering of the DROIDs over pixel arrays, where an equivalent 1024 (32×32) pixel array would need to dissipate ~ 300 nW, significantly greater than the capability of these mounts.

Table 6.8. Fitted parameters to the power plateau measurements of the detector mounts as shown in Figure 6.5.

Parameter	Device 1	Device 2
T_C	92.4 ± 3.3 mK	$98.4 \pm 2.4 \text{ mK}$
K	$39.7 \pm 2.3 \ \mu W \ K^{-n}$	$20.3\pm1.0~\mu W~K^{\text{-n}}$
п	3.82 ± 0.06	3.92 ± 0.05
$G(T_C)$	$184 \pm 22 \text{ nW K}^{-1}$	$89 \pm 8 \text{ nW K}^{-1}$
G (177 mK)	1.1 μW K ⁻¹	0.5 μW K ⁻¹



Figure 6.5. Power measurements for SRON devices on Si substrate.

The heat flow from the TES to the Si₃N₄ membrane will be dominated by either the Kapitza resistance or by electron-phonon coupling. The Kapitza resistance is a thermal boundary resistance at an interface between two different materials. This Kapitza resistance is described by an acoustic mismatch model, where some of the phonons are reflected off the interface and some are transmitted [Swartz and Pohl, 1989]. In this régime the power radiated across a contact area *A*, between two materials at different temperatures, obeys a Stefan-Boltzmann law. Thus in Equation 2.9, $K = A\sigma$, where σ is the Kapitza conductance and n = 4. Roukes et al. (1985) estimate $\sigma = 125$ W K⁴ m² for Cu on a sapphire substrate. This value is assumed here in the absence of other experimental data. We can then estimate the thermal conductance at the TES-Si₃N₄ interface due to the Kapitza resistance G_{kap} , which, from Equation 2.10 is $G_{kap} = 4\sigma AT^3 = 173$ nW K⁻¹. Given our own experimental measurements, which gives $G \sim 30$ times smaller than G_{kap} , it is unlikely that Kapitza conductance is the dominant transport mechanism in these DROID devices.

The measured thermal exponent of n = 4.1 for the single pixel Ir/Au bilayer TES (from a different chip to the DROID devices – Table 6.7) is however consistent with that expected for a Kapitza resistance dominated process. Assuming $\sigma = 125$ W K⁴ m², we can again estimate the Kapitza Conductance, $G_{kap} = 4\sigma AT^3 = 10.5 \pm 3.4$ nW K⁻¹; several times greater than measured. Conversely, assuming the measured value of *G* from Table 6.7, we can estimate $\sigma = 32.6 \pm 21.6$ W K⁴ m², which is several times smaller than the assumed value of σ . Further measurements with better statistics and using different bilayer devices are required to accurately estimate the Kapitza conductance between the Si₃N₄ and Ir interface. At mK temperatures, electrons in the TES may be driven far out of equilibrium with the phonon distribution. In this scenario the electrons will transfer heat to the phonons at a rate governed by Equation 2.9, with $K = \Sigma V$, where Σ is the electron-phonon coupling parameter, V is the detector volume and typically n = 5-6 [Wellstood et al., 1994]. If the phonon wavelength is longer than the TES thickness no discrete phonon population can exist in the TES, consequently, the substrate and TES phonons are coupled across the interface existing as a single population. Here, any Kapitza boundary resistance becomes vanishingly small and electron-phonon coupling will dominate the heat transfer from the TES to the substrate. Using an estimate for the electron-phonon coupling parameter $\Sigma \sim 1 \times 10^9$ W m⁻³ K⁻⁵ [Chapellier, 2004] and taking n = 5, makes $G_{e-ph} = 5\Sigma VT^4 = 31$ nW K⁻¹. Our experimental measurements are not consistent with electron-phonon coupling as the dominant transport mechanism in our TES DROIDs at the temperatures investigated. Kapitza conductance and electron-phonon coupling are more likely to become more important at lower temperature because of their higher thermal exponent n. We have small test devices for measuring the electron-phonon coupling parameter in Ir, which we aim to test in the near future.

Leivo and Pekola (1998), investigate thermal conduction in Si₃N₄ membranes and show that the membrane thermal conductivity (κ) can be described by $\kappa = 14.5T^{1.98}$ mW m⁻¹ K⁻¹, which would make n = 2.98. These authors also quote data by Holmes where $\kappa = 16.2T^{2.5}$ mW m⁻¹ K⁻¹ thus making n = 3.5. Hoevers et al. (2005) report a consistent value of n = 3.6 for their Si₃N₄ membranes in various geometric configurations. Our measured values of n, ranging between 2.63 and 3.40, would seem consistent with this published data for thermal transport dominated by the Si₃N₄ membrane.

Practically, the thermal conductance to the cold bath can be adjusted (for different count rate and energy/position resolution requirements) by varying the dimensionality of the membrane, the positioning of the TES on the membrane or by tuning the transition temperature.

6.5. Summary

We have already discussed the importance of the absorber thermal conductance G_{abs} and the thermal conductance to the bath G, in DROID design, in particular their ratio g, for optimising both energy and position resolution (Chapter 4). In future DROID designs we will investigate Cu as the thermal diffusion layer as an alternative to Au because of the improved absorber conductance for a given heat capacity. We have shown how the heat flow from the TES to the cold bath is consistent with thermal transport limited by the Si₃N₄ membrane. Further design and characterisation of the Si₃N₄ membranes is required in order to optimise G and hence the energy and position resolution (though this will depend ultimately on the application and required science goals). We will also pursue Ir/Au bilayer technology for use as DROID TESs rather than the monolayer Ir films due to their lower T_C , R_n and

improved thermalisation. For longer term array development we will be investigating improved coupling schemes to maintain low stress on the membrane during thermal cycling, coupled with a high conductance between the mount and cold bath.

Chapter 7: TES Current-Voltage Characterisation and X-ray Testing

This Chapter describes the initial experimental testing of one of our Ir TES DROIDs and a single pixel Ir/Au TES. These measurements include the current-voltage characteristics of the TESs as well as the first X-ray testing from both a DROID and single pixel TES. Some of the results in this Chapter are reported in [Smith et al., 2004; 2006c].

7.1. DROID Iridium TES

7.1.1. Current-Voltage Characterisation

The current-voltage (*I-V*) curve for a TES defines the normal-to-superconducting transition region of the device. This curve can be used to calculate the power, resistance and temperature of the device throughout the transition region, from which, estimates of key parameters such as G, α and n can be made.

The top loading 3 He/ 4 He Dilution Refrigerator (DR) (see Section 5.3.3) was used to cool the DROID Ir TES to below its superconducting transition, measured to be ~ 177 mK. The first device investigated was device 112/2 DROID 2 TES 1, as listed in Table 6.5, deposited half on the membrane and half on the substrate (see Figure 6.2). TES 2, at the other end of the DROID, was not connected to any bias circuitry. The device was voltage biased using the on-chip 35 m Ω shunt resistor and a large series resistor of typically ~ 10 k Ω . A DC voltage offset was supplied using the Oxford Instruments bias controller; an AC component (in the form of a triangular wave) was superimposed on the DC signal to sweep through the transition edge at a rate of 20-50 mHz. An Oxford Instruments SQUID and amplifier chain (see Section 5.1) was used to measure the current through the TES and the data was logged using the in-house-built ADC card and software at a sample rate of 1 kHz. *I-V* curves at different base temperatures were measured using the mixing chamber heater to vary the DR temperature whilst adjusting the voltage bias to keep the device biased within the transition edge. A ⁵⁵Fe X-ray source was used to irradiate the whole detector area with 5898 and 6490 eV photons (see Section 5.4.1).

To derive the *I-V* curves the measured SQUID output was scaled to the TES input current using the normal, straight-line part of the *I-V* curve, where the resistance is known to be 0.75 Ω . The input voltage was then converted into a current through the bias circuit; the assumption here is that the series resistance is very much bigger than the TES-shunt combination so the latter can be ignored. The resistance of the TES and hence the voltage across it, are found from the ratio of the input circuit current to the TES current (knowing that the voltage across the TES and the shunt are equal). The power dissipated in the TES follows from the Joule term I^2R .

Figure 7.1 shows a set of *I-V* curves for six different DR base temperatures. These *I-V* curves take the anticipated shape for a voltage biased TES. The straight-line region corresponds to the TES in the normal state and as expected, the transition-edge region leads to a negative differential resistance (dV/dI). The data shows a shift in voltage bias when the base temperature is changed; this follows from the fact that as the base temperature is raised, less Joule power and hence applied voltage is required to bias the device (see Equation 2.9). These data are consistent with the power plateau measurements described in Section 6.4.



Figure 7.1. I-V curves for an iridium DROID TES at different base temperatures. The inset shows a zoom in of the step feature observed for $T_b = 11$ mK. X-ray pulses can be seen in the I-V curves as small negative going current spikes in the transition region.

Figure 7.1 shows a small step feature in the *I*-*V* curve (~ 220 nA) towards the top edge of the transition region, which occurs for all bath temperatures, becoming less discernible for temperatures close to T_C .

This feature is present in all data. Careful examination shows that the transition curve is in fact a different shape before and after the step (in other words it is not just a discrete step but a shape variation in the transition itself). Furthermore, when the TES is biased below the step, some of the observed X-ray pulses in fact exhibit oscillations or step-like features (see Section 7.1.2). It was also noticeable that when increasing the bias voltage beyond this first step, several successive smaller steps became apparent (exhibiting hysteresis), until the normal state was finally reached. This is demonstrated by the inset in Figure 7.1, where to limit noise, successive data samples were averaged on a digital oscilloscope and then both the input and output traces were saved to file and recombined post facto.

Such structure in *I-V* curves complicate the operation of a TES and requires an understanding of the underlying physics involved so that they can be removed. The physics of the resistive state (on the transition-edge) for thin films is complex and is thought to depend upon whether the film is Type I or Type II as well as the dimensionality of the system. Steps in the *I-V* curves of superconductors have been observed for thin-films, micro-bridges and filamentary crystals (whiskers) [Skocpol et al., 1974a; Huebener and Gallus, 1973; Huebener and Watson, 1794; Webb and Warburton, 1968]. Wollman et al. (1997) observed steps in the *I-V* curves for their Al/Ag TES microcalorimeter which they removed by the application of a small magnetic field.

For one-dimensional, Type-I, superconducting systems such as whiskers or thin-film micro-bridges (where the transverse dimension $w \leq \xi(T)$ (the Ginzburg-Landau coherence length) and also $\leq \lambda(T)$ (the penetration depth)), it has been shown that step features in *I-V* curves can arise due to quantum phase slip centres (PSC's), where the local critical current has been exceeded [Skocpol et al., 1974a]. For three dimensional systems of thickness d (w, $d \gg \xi(T)$, $\lambda(T)$), step structure is associated with the abrupt nucleation of channels of flux tubes of opposite phase, originating from the edges of the film, moving towards the centre and annihilating [Huebener and Kampwirth, 1972]. It is argued that a TES detector can be considered a two dimensional system with $w < \lambda$ (~ 10's cm [Fraser, 2005] and $d < \xi(T)$ (for Ir $\xi_0 \ge 330$ nm [Trowell, 2001])). In this régime the analogue to the one-dimensional PSC is the quantum phase slip line (PSL).

It has been reported that such effects become less observable when $T_b \ll T_C$ [Schulze and Keck, 1983; Skocpol et al., 1974b] because the PSCs turn into resistive hotspots when the power dissipation becomes larger at lower base temperatures. However, Huebener and Watson (1974) show how voltage steps in indium microstrips become more distinct at lower bath temperatures. Our own observations appear to agree with this latter pattern, since the step feature apparently fades at base temperatures closer to T_C . Steps in (usually current biased) Type-I superconducting thin films have been well publicised. For voltage biased thin films, thermally isolated from a heat sink, such as TESs, steps have been reported [Wollman et al., 1997] but do not appear to be well studied. These phase slip processes have also been proposed as a source of excess noise in TES detectors (phase slip shot noise), where the independent motion of these quantised flux vortices gives rise to a voltage noise in the TES [Fraser, 2005].

Given the unusual TES configuration, with the differing thermal anchoring of the two halves of the TES and different sized electrodes, it is perhaps not surprising that unusual features in the *I-V* characteristics are present. We should note that similar testing of the Ir/Au bilayer TES (chip 113/1) revealed no such step feature (see Section 7.2.1). This TES is again in the "½ on, ½ off" configuration but the electrodes are of equal length.



Figure 7.2. Power P in the TES as a function of the TES bias voltage V, for iridium DROID TES at different base temperatures.

Figure 7.2 shows the voltage-power (*V-P*) curves derived from the data in Figure 7.1. The X-ray pulses and step feature follow through into this data set, where both events are associated with a drop in Joule power due to a sudden temperature rise. As is expected the power flattens out in the transition region – though there is still a slight gradient – before rapidly increasing in the normal state. Rearranging Equation 2.9 which governs the heat flow from the TES to the cold bath, we can make an estimate of the TES temperature *T* as a function of resistance *R* throughout the transition edge:

$$T = \left(\frac{I^2 R}{K} + T_b^n\right)^{1/n}.$$

Using Equation 7.1 we assume that both K and n (and therefore G) are constant throughout the transition edge, although from Section 6.4, we found evidence that these parameters are different at the top and the bottom edge of the transition. We use the data in Table 6.6 evaluated at the very top of the transition. This fixes the top edge of the transition at $T_C \sim 177$ mK but we should note that the data in Table 6.7, evaluated at the bottom of the transition, shows different K, n and G, and would result in a different R-T transition. Figure 7.3 shows the R-T data derived using Equation 7.1, for the different base temperatures investigated. The sharp step features in the R-T plots are physically unrealistic in this context because of the assumptions in the use of Equation 7.1. These step features results from an increase in temperature, causing in a drop in current and therefore Joule power, not an increase in current as assumed by Equation 7.1.



Figure 7.3. R-T curves for iridium DROID TES at different base temperatures. Both the resistance and temperature data are smoothed with a 9-point moving average to remove high frequency noise.

The transition edges show an apparent strong broadening with decreasing base temperature (increasing current). A degree of current dependence is expected from the theory described in Section 2.2.2 and is consistent with other authors' findings [Tan et al., 2002; Frank et al., 1994]. As well as current dependence, *R-T* transitions are in general broadened as a result of edge effects [Hilton et al., 2001], ambient magnetic fields [Ullom et al., 2004; Takei et al., 2004] and vortex pinning at impurity sites.

However, it is likely that the observations here are more associated with the assumption in the use of Equation 7.1, namely the assumption that G is constant throughout the transition.



Figure 7.4. TES sensitivity α as a function of bias voltage V for the six different base temperatures investigated. The key is the same as for Figure 7.3.

Figure 7.4 shows the calculated α values as functions of TES bias voltage for the six different base temperatures, confirming that the broadening transition with decreasing base temperature increases α substantially. Further measurements are required under constant current bias conditions to identify the true extent of any current dependence in the transition region.

7.1.2. DROID X-ray Testing

Due to the availability of a single SQUID at the time, it was only possible carry out single TES readout of a prototype DROID 2. On the initial experimental run, data was logged in a continuous stream using the in-house-built ADC (see Section 5.2) at sampling rate of 160 kHz. These initial results are reported elsewhere [Smith et al., 2004]. We repeated this analysis on data gathered on a second experimental run, with the improved fridge wiring (see Section 5.3.3). The detector was biased at various points on the transition edge and data gathered in continuous streams for up to 200 s at an ADC sample rate of 416 kHz. The data reported below is a typical set gathered at $R_0 \sim 0.55 \Omega$ on the transition edge. We have used a box car (top hat) function to filter the raw, continuously logged data in the time domain, to locate peaks corresponding to X-ray pulses. The baseline is subtracted and sections of data with peaks above a threshold level are stored sequentially in an array, 2750 samples long (6.6 ms). Sections of data with double peaks in the data are rejected in an attempt to limit the effects of pile up. In this particular data set 6670 pulses were identified. To extract spectral and spatial information we need to apply the optimal/matched filtering described in Section 4.1.1. For simplicity, we work in the time domain with the assumption that the noise is white. Then, a filter template F(t) can be created by averaging together all the detected pulses,

$$F(t) = \overline{D(t)} + O.$$

Where O is an offset added to the filter so that the integral over time is zero. This has the effect of removing baseline variations throughout the data set. Figure 7.5 shows the average filter template F(t).



Figure 7.5. Matched filter defined using the average of all the detected 6670 pulses.

This filter template is then convolved with the raw data, with a sequentially increasing time index *i*, to allow for any trigger jitter,

$$H_i = D(t_0) * F(t_i).$$
 7.3

The maximum of these convolution responses over i is then used as the estimate of the pulse amplitude. Equation 7.3 is the time domain equivalent of Equation 4.4 under the assumption of white noise. This process is repeated for each detected pulse. A pulse amplitude distribution is then generated as shown in Figure 7.6. This shows that most of the pulses are small amplitude with a rapidly decreasing population as the amplitude is increased.



Figure 7.6. Pulse height distribution using the filter template shown in Figure 7.5.

We can investigate the different pulse shapes in this distribution by averaging sets of pulses in different amplitude bins. Figure 7.7 shows two different pulse shapes found by averaging pulses with amplitudes between 10-22 ($F_a(t)$) and 75-125 ($F_b(t)$). Thus we can see that the distribution contains pulses of different amplitude and of different shape (rise/fall time).



Figure 7.7. Average pulse shapes. The black curve is $F_a(t)$ and the red curve is $F_b(t)$.

To investigate the populations of different pulses further, the data is then re-filtered with both these new matched filters using the same method as described above and the peak amplitude recorded for each: H_a and H_b respectively. We then define the ratio of the estimated pulse amplitudes as the *shape* parameter, Q:

$$Q = \frac{H_a}{H_b} \,.$$

This shape parameter gives an indication of which template filter ($F_a(t)$ or ($F_b(t)$) a particular pulse is more closely matched to and is a method of distinguishing between different populations of pulses. Figure 7.8 shows the shape parameter Q versus pulse amplitude H_a . The pulses with a smaller Q value are wider (have a longer decay time) than those with a larger Q value.



Figure 7.8. Shape parameter Q as a function of H_a . The main sequence of pulses corresponding to absorption in the Au absorber layer is indicated, along with the estimated positions at x = 0 and x = 1/2.

The average of all the pulses in the blue dashed box and the red dotted box are shown in Figure 7.9a,b. The largest amplitude pulses have to correspond to direct absorption in the TES, where the maximum fraction of energy is detected by the sensor. The energy of the pulses can be estimated by integrating their area over time, using Equation 2.36. The energy of the blue pulse is then estimated to be ~ 4.8 keV compared to the X-ray photon energy of 5.9 keV. Recall that only in the extreme ETF limit with perfect voltage bias will the photon energy equal the measured energy. The small red pulses in Figure

7.9a, have an energy estimated to be ~ 0.1 keV, significantly less than the incident photon energy. These small, fast pulses are likely to be absorbed in the 375 μ m thick Si substrate below and around the TESs, or in the 500 nm thick Si₃N₄ membrane, where most of the energy escapes directly to the heat bath and only a small fraction is detected by the TES.



Figure 7.9. a) Average pulse shape taken using the data in the red dotted box in Figure 7.8 and b) average pulse shape taken using the data in the blue dashed box in Figure 7.8.

The bottom band of pulses in Figure 7.8 (defined as the "main sequence") corresponds to X-rays absorbed in the Au absorber. As predicted by the DROID theory in Chapter 3, the largest, fastest pulses are absorbed close to the TES (at x = 0 as indicted on Figure 7.8). As the distance from the TES is increased, the pulses become smaller and slower (due to the low pass filter effect of the absorber conductance), resulting in a single peaked pulse height distribution at the low end of the amplitude scale.

The two thick position calibration blocks on the DROID absorber at relative positions x = l/2 and x = 3l/4 (see Figure 6.2), permit some degree of position determination even when reading out the DROID with a single TES. These thicker blocks will absorb approximately twice as many counts per unit area compared to the rest of the absorber, thus two peaks should stand out in a pulse height distribution due to differential absorption at these points. Figure 7.10 shows the pulse height distribution only for events thought to be absorbed in the Au layer. Clearly visible are two peaks in the distribution on top of a general distribution, which as expected, is weighted to the low end of the amplitude scale. The smaller peak is likely to correspond to the first calibration point at x = l/2 and the second peak is likely to include the effect of the second calibration point at x = 3l/4, as well as incorporating the increasing population of low amplitude pulses with increasing *l*. Thus the position sensitivity of the prototype TES DROID is demonstrated for the first time.



Figure 7.10. Pulse height distribution for Au absorber pulses. Marked on the distributions are the positions thought to correspond approximately to x = 0, l/2, 3l/4.

Figure 7.11 shows average pulse shapes taken along this main sequence in bin widths of $0.5H_a$, between $H_a = 0$ and 6. Figure 7.12 shows a semi log plot of the same data. These data show the expected signal variation with position and are consistent with the simulated pulse shapes and the basic theory developed in Chapter 3 (see Figures 3.5 and 3.6). Significantly, these data confirms the spectral and spatial resolving capabilities of the DROID concept to X-ray photons. However, given the complex nature of the DROID and the number of the variables which determine the measured signal at the output, it is not possible to accurately fit the single channel data to the modelling developed in Chapter 3, for the 'ideal' dual channel read out DROID.

The maximum energy of the absorber pulses is estimated to be ~ 4 keV corresponding to $x \approx 0$, whereas the energy at $x \approx l/2$ drops to less than half that value to ~ 1.3 keV. Thus it is likely that some of the energy is being lost from the Au absorber to the Si₃N₄ membrane and escaping to the heat bath without being detected by the TES. In future DROID designs we will cut the membrane either side of the absorber parallel to it, ensuring that any phonons emitted from the absorber will be channelled towards the TESs, or re-absorbed in the Au layer, rather than escaping direct to the heat bath. The largest X-ray pulses are typically less than 1 µA in height and more commonly, some 100's nA; this implies a maximum resistance change of just $\Delta R \sim 30$ m Ω or < 4 % of the total dynamical range of the TES.



Figure 7.11. Pulse shapes along the "main sequence" averaged in $0.5H_a$ bin widths. The largest fastest pulse corresponds to pulses in the interval $H_a = 5.5-6.0$ and the smallest slowest pulse to the interval $H_a = 0-0.5$.



Figure 7.12. Semi-log plot of the data shown in Figure 7.11. After spatially variant thermalisation, the pulses decay with approximately the same exponential time constant of $399 \pm 57 \mu s$.

These small pulse heights indicates that the ratio of C/α is much bigger than E/T_0 (see Equations 2.44 and 2.45) and could be reduced by approximately a factor of 10, which could improve the resolution by $\sim \sqrt{10}$ (see Equation 2.43) without compromising the saturation conditions. The most obvious way to achieve this is by reducing the heat capacity of the DROID absorber, either by material choice (see Sections 6.1 and 6.2) or changing the geometry.

One data set was gathered at $R_0 = 0.66 \Omega$ ($V_0 = 14.1 \mu$ V), just below the step feature found in the TES *I-V* curve shown in Figure 7.1. Pulses with peaks amplitudes ≤ 300 nA took the expected form, similar to those shown in Figure 7.11. Pulses with peak amplitudes ≥ 300 nA, whether absorbed in the TES or the absorber, were all observed to have a peak amplitude of ~ 1 μ A with a ~ 300 nA step feature on the decay, followed by a negative undershoot. Figure 7.13 shows the average of these pulse shapes. These anomalous features illustrate the operational difficulties in the presence of step-structure in the transition edge.



Figure 7.13. Average pulse shapes for pulses of peak amplitude > 300 nA when the TES is operated at $R_0 = 0.66 \ \Omega$. The anomalous step feature in the pulse decay and the negative current undershoot are clearly visible.

7.1.3. Noise and Energy Resolution

To estimate the energy resolution we can bin up a section of data (1 ms), containing no X-ray pulses. This baseline noise has an amplitude distribution which can be fitted with a Gaussian function, from which, the full width at half maximum (FWHM) can be calculated. Assuming the largest X-ray pulse height in the data corresponds to a 6.49 keV photon, we can make an approximate estimate of the

baseline energy resolution. On the first experimental run the baseline noise was estimated to be ~ 150 eV, similar to a Charge Coupled Device (CCD). This level of noise was significantly above the predicted 10-20 eV energy resolution (see Chapter 4) and was a result of the non-optimum wiring in the DR (see Section 5.3.3), inadequate grounding and non-optimum pre-amplifier (see Section 5.1.5). Optimisation of the DR internal wiring harness, as described in Section 5.3.3 improved the baseline noise by approximately a factor of ~ 3. Table 7.1 shows the estimated baseline noise as a function of bias point on the transition edge. The estimated energy resolution is approximately 50-60 eV throughout the transition. The measured white noise level, estimated at a frequency of 1 kHz, in the transition region is shown on Figure 7.14. Also shown is the calculated noise level using the model developed in Chapter 3, considering only noise sources associated with TES 1 and the absorber phonon noise. Finally, we show the quadrature sum of the theoretical noise and the noise contribution estimated from the read-out chain (assumed to be ~ 22 pA/\Hz, see Table 5.2).

Table 7.1. Baseline energy resolution ΔE at the different operating resistances R_0 . Data at R = 0.66 Ω is omitted because of the anomalous pulse shapes.

$R_0(\Omega)$	R/R_N (%)	ΔE (eV)
0.24	32	64.3 ± 1.4
0.33	44	50.7 ± 1.1
0.46	61	55.5 ± 1.2
0.55	73	54.2 ± 1.1

The measured noise is likely to be dominated by the amplifier noise though there is evidence that there may be an additional amount of unexplained (excess noise) possibly associated with the TES. High quality noise spectra obtained with the improved read-out electronics (see Section 5.1.5) is now required both on the transition edge and in the normal/superconducting states, to understand whether the theoretical intrinsic detector noise limit is attainable, or whether there is unexplained noise present.



Figure 7.14. Comparison of theoretical detector noise (squares), the quadrature sum of amplifier and detector noise (triangles) and the measured noise (diamonds). The data is taken at ~ 1 kHz where the noise is approximately white.

7.2. Iridium / Gold Bilayer TES

7.2.1. Current-Voltage Characterisation

We also tested the Ir(100)/Au(210) bilayer TES on chip 113/1 in the top loading DR at the Cavendish Laboratories, using the same experimental method as described above. An optimised SQUID and amplifier were used (read-out chain 2, as described in Table 5.2) provided by the Cavendish Laboratories (Dr. D. Goldie) and the detector was biased using a constantan shunt with $R_S \sim 1 \text{ m}\Omega$. The *I-V* curve for this device was is shown in Figure 7.15, with the inset showing the power in the transition edge, both plots are derived using the same methods outline above. We see that the *I-V* curve does not apparently show the step feature that was found in the Ir DROID TES transition. Fine structure in the transition is illustrated in the *R-T* curve shown in Figure 7.16, from which $\alpha(T)$ is estimated (inset of Figure 7.16). The relatively broad transition results in $\alpha(T)$ being quite small (< 40) for the majority of the transition with some higher peaks at the top edge. The long resistive tail is most likely a result of the increasing current at lower bias points. Measuring the transition edge under constant current bias conditions we should find that the shape is constant but the transition temperature changes [Tan et al., 2002]. T_C decreases as current increases, consequently when R(T) measured under constant voltage bias conditions, the transition is broadened.



Figure 7.15. I-V curve for Ir/Au TES at $T_b < 10$ mK. The inset shows the power through the transition.



Figure 7.16. *R*-*T* transition for Ir/Au TES at $T_b < 10$ mK. The data is smoothed with a 7-point moving average over both R and T to remove high frequency noise. The inset shows the logarithmic sensitivity α , as a function of TES temperature.

Figure 7.17 shows the loop gain L_0 estimated throughout the transition edge. Because of the small sensitivity (α), $L_0 < 20$ at all bias points reflecting the effect of the ETF (see Section 2.2 for the implications of L_0 and ETF on detector performance).



Figure 7.17. The loop gain L_0 as a function of TES resistance derived from the data in Figure 7.16.

7.2.2. X-ray Testing

Limited X-ray testing was carried out on the Ir/Au TES using a ⁵⁵Fe source (provided by the Cavendish Laboratory) mounted in close proximity to the TES. The count-rate was disappointing and estimated to be < 100 mHz. Unfortunately there was also a problem with the electronics, which introduced a low pass filtering effect which limited the bandwidth to ~ 20 kHz, consequently the rise time was limited to ~ 18 µs (Equation 5.11). Due to limited slew rate capabilities of the read-out electronics and the large amplitude of the pulses, it was not possible to maintain flux lock for sufficient periods to gather reasonable X-ray data, when biasing at the low edge of the transition (where the bias current is higher). A small amount of X-ray data (~ 50 pulses) was gathered at a high bias point 83 m Ω (0.85*R*_N) in the transition. At this point in the transition, from Figure 7.16 we estimate $\alpha \approx 50$, similarly the heat capacity of the Ir/Au TES is approximately 0.5 pJ K⁻¹ at *T_C* = 76 mK and from the thermal analysis, *G* ~ 2.8 nW K⁻¹, therefore we can estimate $\tau_0 \approx 179$ µs and $\tau_{eff} \approx 14$ µs, compared to a measured $\tau_{eff} \approx 36$ µs. Using Equation 2.44 substituting for *R*₀ $\approx 0.85R_N$ and $\Delta R \approx 0.25R_N$, re-arranging for the saturation energy we find that $E_{sat} \approx 1.4$ keV, thus photons from the ⁵⁵Fe X-ray source (5.9 keV) were several times more energetic than this and consequently saturated the TES. Thus it was not possible to obtain spectral resolution with the data gathered.



Figure 7.18. Average pulse shape for X-ray events in the TES. The inset shows a semi-log plot of the same data. The anomalous shape is due to the limited bandwidth of the read-out chain and the effect of saturation in the TES.

Figure 7.18 shows the average pulse shape of all the TES hits, clearly shown is the rounded top of the pulse shape, indicative of saturation. If it were possible to bias at $0.2R_N$ where $\alpha \approx 10$, then we find $E_{sat} \approx 90$ keV and the device should not saturate. Using the Equation 2.43 to estimate the best theoretically achievable FWHM energy resolution, we find $\Delta E \approx 2.1$ eV. Thus despite the limited quality data gathered in this run it is worth testing this device in the future with optimised read-out electronics, since good performance is theoretically achievable.

7.3. Summary

In this Chapter we have reported the first experimental X-ray testing of a prototype DROID device, confirming its spectral and spatial resolving potential (consistent with the theory developed in Chapters 3 and 4), though the full position and energy resolutions with dual channel read-out have not yet been measured. Testing a DROID 1 device with dual channel read-out using the optimised read-out chain described in Section 5.1.5 is in preparation and planned for the near future using the side arm fridge at Leicester. The anomalous features in the DROID 2 TES *I-V* data require comparison with other devices in order to further understand and eliminate such features. The testing of the Ir/Au bilayer TES is only very preliminary but is an alternative lower T_C option than the monolayer Ir TES.
Chapter 8: Conclusions and Future Research

This Chapter briefly summarises the main conclusions of the work reported in this thesis and outlines a number of future research aims.

This thesis has described the theoretical and experimental development of Transition Edge Sensor Distributed Read-Out Imaging Devices for use in future X-ray astronomy observatories. The linear model developed in Chapter 3 has given a detailed understanding of how the noise correlations between the two TESs in the DROID impact on performance. This understanding, led in Chapter 4, to the development of optimised signal processing algorithms for position and energy determination. We have carried out detailed numerical simulations to evaluate energy and position resolution across typical DROID designs, using pre-existing methodologies and the new optimal filtering algorithms. These algorithms provide, for the first time, a basis for calculating the truly optimised energy and position resolution, based on the intrinsic noise characteristics of the DROID. We have shown that these new algorithms can result in improved energy resolution and particularly position resolution, compared with existing published techniques, though further work is required to extend the analysis to include non-linear effects (such as saturation at high photon energy). The simulations also reveal the importance of the DROID thermal conductances and heat capacities in optimising both the energy and position resolution using the new signal processing algorithms. In particular, we have shown the importance of the ratio g, of the absorber thermal conductance G_{abs} to the thermal conductance to the heat bath G, on both the energy and position resolution and the trade-off between the two. This analysis provides a complete theoretical framework for the design of practical DROIDs and is an important step forward in the development of position-sensitive detectors. This analysis is equally applicable to other position-sensitive technologies and as such, we are working to extend the theory to investigate Superconducting Tunnel Junction (STJ) and Metallic Magnetic Calorimeter (MMC) DROIDs.

The development of DROIDs at Leicester will continue to be targeted at the European Space Agency's XEUS mission. Although DROIDs can be designed to offer comparable energy, position and count rate capabilities, they should not be viewed as a replacement of the proposed pixel array focal plane instrument for XEUS but as a possible extension to it, increasing the focal plane coverage of the

cryogenic narrow field instrument (NFI). In addition, large absorber, DROID type detectors are potentially well suited for a large sparse array as is required in the development of a practical X-ray interferometer (which requires an energy resolution of < 100 eV coupled with a position resolution of ~ 10-30 µm) [Willingale, 2004]. The practical implementation of the signal processing algorithms to devices with differing design parameters (such as the *g* ratio), for these different applications, may require differing approaches. The fabrication and testing of new DROIDs designed over a range of parameters will be required to fully investigate the effectiveness of the algorithms and develop a suitable implementation scheme. For example, good ΔE with moderate Δx is typically required for missions such as XEUS, which in turn implies devices fabricated with $g \sim 100$. For applications such as X-ray interferometery, which require moderate ΔE with good Δx ; devices with $g \leq 1$ are more suitable. The key to the success of the theory is whether fast and efficient algorithms can be implemented in real time, and is likely to depend on the amount of effort put into device calibration, as well as computation requirements.

At the time of submission, by moving to a faster computer system, we have now been able to improve the accuracy of the model by increasing the number of absorber elements from 21 (as used for calculations in the thesis) to 80, without increasing the computational time. We are using this extended model to develop suitable implementation schemes for use on real data. Essential to this is the gathering of good quality noise and X-ray data in dual channel operation. In the longer term we will use the extended model to investigate other absorber geometries, such as four TESs on a single large rectangular absorber (two-dimensional imaging spectrometer) for example.

Chapter 5 is devoted to the description of the read-out and cooling requirements for the operation of TES detectors. The characterisation of the SQUID and its associated read-out electronics used in the experimentation reported in the thesis is described. All testing to date has been carried out in the top loading dilution refrigerator. The recently refurbished Leicester side-arm refrigerator is now in-place in the purpose built cryogenics laboratory in the Space Research Centre and will be used in all future testing.

The fabrication of the experimental DROIDs reported in Chapters 6 and 7, pre-date the theoretical understanding developed within Chapters 3 and 4. These devices were designed using the approximation that the energy and spatial resolving powers are of the same order ($\Delta E/E \sim \Delta x/L$); an approximation that we have shown to be inadequate for a complete description of the device performance. The material characterisation discussed in Chapter 6 and the preliminary X-ray testing reported in Chapter 7, confirm that these current devices are not well optimised for the XEUS mission. However, we have now demonstrated the capability to optimise future DROID designs by adjusting material selection and geometry, in conjunction with the implementation of the new filtering algorithms. The choice of iridium for the TES is not, in retrospect, ideal given the low thermal

conductivity and the likelihood of phase separation; lower resistance, lower transition temperature Ir/Au bilayers will be pursued in future designs. Measured current-voltage characteristics for an Ir DROID TES reveal anomalous step features in the transition edge, which do not appear in the similar Ir/Au transition data. We require comparative data from other Ir TES configurations in order to further understand and eliminate such features. Noise data is also required for both Ir and Ir/Au technologies to evaluate the degree to which these TESs may exhibit unexplained 'excess' noise.

The first X-ray results from a prototype DROID 2 configuration, although with only single channel read-out, have demonstrated proof of concept, confirming that position sensitivity has been achieved. Despite the estimated baseline noise of \sim 50-60 eV being somewhat worse than is expected by the theoretical simulations (< 22 eV), largely due to poor quality read-out electronics, improved performance is expected using the optimised amplifier chain described in Section 5.1.5. Dual channel read out X-ray testing of both DROID configurations is currently being arranged in the side access refrigerator at Leicester, with results expected in the near future. These results are essential for comparing the differences between the performance of the DROID 1 and 2 configurations and comparing the overall capabilities (energy resolution, position resolution and count rate) of these devices with the theoretical simulations of Chapters 3 and 4.

Appendix A: Calculation of Variances and Covariances

In this appendix we derive the variances used in Chapter 4, which describe the energy and position resolution along the length of the DROID and the covariance between them (Equations 4.33, 4.34 and 4.35). The starting point for these derivations is Equation 4.31:

$$\begin{pmatrix} H \\ H\delta x \end{pmatrix} = \left[\int F^T S df \int F^{/T} S^{/} df - \left(\int F^{/T} S df \right)^2 \right]^{-1} \left(\int F^{/T} S^{/} df - \int F^{/T} S df \right) \left(\int F^T D df \right)^2$$

$$4.31$$

where *H* is the energy estimate and $\delta x = (x_{\gamma} - x)$ is the difference between the actual interaction position x_{γ} and an initial estimated position *x*. The vectors *D* and *S* are column matrices: $D = \begin{pmatrix} D_1(f) \\ D_2(f) \end{pmatrix}$, $S = \begin{pmatrix} S_1(f) \\ S_2(f) \end{pmatrix}$ containing the measured data and the signal responsivity respectively, at each TES and $F = \begin{pmatrix} F_1(f) \\ F_2(f) \end{pmatrix}$ is the matched filter matrix containing the filters to apply to each TES signal. Superscript *T* terms are matrix transposes and primed terms at differentials with respect to *x*.

Equation 4.31 can be re-written in the form:

$$\begin{pmatrix} H \\ H\delta x \end{pmatrix} = [ca - b^2]^{-1} \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix},$$
 A1

so that, $H = [ca - b^2]^{-1}(aX + bY)$, and $H\delta x = [ca - b^2]^{-1}(bX + cY)$.

From Equation A1 the variance on *H* can be found as follows:

 $\sigma_H^2 = Var(H)$

$$= \left[ca - b^{2}\right]^{-1} Var(aX + bY)$$

$$= \left[ca - b^{2}\right]^{-1} \left\langle \left((aX + bY) - (a\overline{X} + b\overline{Y})\right)^{2} \right\rangle$$

$$= \left[ca - b^{2}\right]^{-1} \left\langle a^{2} (X - \overline{X})^{2} + b^{2} (Y - \overline{Y})^{2} + 2ab(X - \overline{X})(Y - \overline{Y}) \right\rangle.$$

Here, $(X - \overline{X})^2$ is the variance on X, σ_X^2 ; similarly $(Y - \overline{Y})^2$ is the variance on Y, σ_Y^2 ; and then $(X - \overline{X})(Y - \overline{Y})$ is the covariance between X and Y, σ_{XY} . Therefore:

$$\sigma_{H}^{2} = \left[ca - b^{2}\right]^{-1} \left(a^{2} \sigma_{X}^{2} + b^{2} \sigma_{Y}^{2} + 2ab \sigma_{XY}\right).$$
 A2

Similarly, the position resolution is found from the variance on $H\delta x$. Here we assume H and δx are dependent variables and we neglect negligible higher order terms in the full expansion of the variance given by Goodman (1960):

$$\begin{split} \sigma_{H\delta x}^{2} &= Var(H\delta x) \\ &= \overline{H}^{2}Var(\delta x) + \overline{\delta x}^{2}Var(H) + 2\overline{H}\overline{\delta x}Cov(H,\delta x) \,, \\ &= \overline{H}^{2}\sigma_{\delta x} + \overline{\delta x}^{2}\sigma_{H} + 2\overline{H}\overline{\delta x}\sigma_{H\delta x} \,. \end{split}$$

As we are looking for the variance when the position has been found, $\overline{\delta x} = 0$ and

$$\sigma_{H\delta x}^2 = \overline{H}^2 \sigma_{\delta x} \, .$$

The variance on δx is then:

$$\sigma_{\delta x}^2 = \frac{1}{\overline{H}^2} Var(H\delta x),$$

so that,

$$\sigma_{\delta x}^{2} = \frac{1}{\overline{H}^{2}} \left[ca - b^{2} \right]^{-1} Var(bX + cY)$$

Using the same methodology used to drive Equation A2 we find,

$$\sigma_{\delta x}^{2} = \frac{1}{\overline{H}^{2}} \Big[ca - b^{2} \Big]^{-1} \Big(b^{2} \sigma_{X}^{2} + c^{2} \sigma_{Y}^{2} + 2ab \sigma_{XY} \Big).$$
 A3

Finally, we need to find the covariance between *H* and δx :

$$\begin{aligned} \sigma_{H\bar{a}\bar{x}} &= Cov(H, \delta\bar{x}) \\ &= \left[ca - b^2 \right]^{-1} H^{-1} Cov((aX + bY), (bX + cY)) \\ &= \left[ca - b^2 \right]^{-1} H^{-1} \langle \left[(aX + bY) - (a\overline{X} + b\overline{Y}) \right] (bX + cY) - (b\overline{X} + c\overline{Y}) \rangle \rangle \\ &= \left[ca - b^2 \right]^{-1} H^{-1} \langle \left[a(X - \overline{X}) + b(Y - \overline{Y}) \right] b(X + \overline{X}) + c(Y + \overline{Y}) \rangle \rangle \\ &= \left[ca - b^2 \right]^{-1} H^{-1} \langle \left[ab(X - \overline{X})^2 + bc(Y - \overline{Y})^2 + ac(X + \overline{X})(Y + \overline{Y}) + b^2(X + \overline{X})(Y + \overline{Y}) \right] \rangle \\ &= \left[ca - b^2 \right]^{-1} H^{-1} \langle \left[ab\sigma_X^2 - bc\sigma_Y^2 + (ac + b^2)\sigma_{XY} \right] \rangle \end{aligned}$$

To solve Equations A2-A4, we need to define σ_X^2 , σ_Y^2 and σ_{XY} .

$$\sigma_X^2 = \left\langle \left(X - \overline{X} \right)^2 \right\rangle,\,$$

which after substituting back for *X* from Equation 4.31 becomes:

$$\sigma_X^2 = \int F^T S df .$$
 A5

Similarly to find σ_Y^2 we have;

$$\sigma_Y^2 = \left\langle \left(Y - \overline{Y}\right)^2 \right\rangle$$
$$= \int F'^T S' df, \qquad A6$$

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and then the covariance between *X* and *Y*, σ_{XY} is:

$$\sigma_{XY} = \left\langle \left(X - \overline{X} \right) \left(Y - \overline{Y} \right) \right\rangle$$
$$= \int F^{T} S df .$$
 A7

Finally, substituting Equations A4-A7 into Equation A2, we have for the variance on *H*:

$$\sigma_H^2 = \frac{\int F^{T} S' df}{\int F^{T} S df \int F^{T} S' df - \left(\int F^{T} S df\right)^2},$$
 A8

similarly from Equation A3 the variance on δx is:

$$\sigma_{\delta x}^{2} = \frac{1}{H^{2}} \frac{\int F^{T} S df}{\int F^{T} S df \int F^{T} S^{T} df - \left(\int F^{T} S df\right)^{2}},$$
A9

and from Equation A4 the covariance between H and δx is:

$$\sigma_{H\delta x} = \frac{-1}{H} \frac{\int F^{T} S df}{\int F^{T} S df \int F^{T} S' df - \left(\int F^{T} S df\right)^{2}}.$$
 A10

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